Appendix to Different cost perspectives for renewable energy support: Assessment of technology-neutral and discriminatory auctions

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1. Model of Cost-based Discrimination

In a RE auction with a given demand D, participate the bidders of two technologies, A and B, with different cost structures, described by the increasing marginal cost functions MC_A and MC_B, where

$$MC_A(x) < MC_B(x) \text{ for all } x \geq 0\,. \tag{1}$$

In the auction, uniform pricing is applied and the price is determined by the lowest rejected bid. Under the assumption that each bidder participates with one project and submits one bid, the auction is incentive compatible, that is, it is optimal to bid the support that exactly covers the costs (**?**). The supply functions are given by

$$S_k(p) = MC_k^{-1}(p),\ k \in \{A, B\}, \tag{2}$$

and increase in the price p. From (1), it follows that

$$S_L(p) > S_B(p) \text{ for all } p \geq MC_L(0)\,. \tag{3}$$

The elasticities of supply of the two technologies are defined as

$$\varepsilon_k(p) = \frac{S_k'(p)}{S_k(p)}p \text{ with } S_k'(p) = \frac{\mathrm{d}S_(p)}{\mathrm{d}p},\ k \in \{A, B\}\,. \tag{4}$$

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In a free competition, the market clearing price p^* is determined by

$$S_A(p^*) + S_B(p^*) = D\,, \tag{5}$$

where $S_A(p^*) > S_B(p^*) \geq 0$. The auctioneer's total support costs are $K(p^*) = p^*D$.

Each of the three discriminatory instruments – quota, maximum price, and bonus – induces a supply shift from the A-bidders to the B-bidders, and different prices p_A and p_B, which lead to the supply volumes $S_A(p_A)$ and $S_B(p_B)$, with

$$S_A(p_A) + S_B(p_B) = D\,. \tag{6}$$

In these cases, the total support costs are

$$K(p_A, p_B) = p_A S_A(p_A) + p_B S_B(p_B)\,. \tag{7}$$

Incentive compatibility holds for a quota Q, which is effective if $Q > S_B(p^*)$, that is, if the B-bidders would not reach Q in a free competition. This leads to a volume shift

$$q = Q - S_B(p^*) \tag{8}$$

from the A-bidders to the B-bidders and to different award prices p_A and p_B, with

$$p_A = MC_A(D - Q) > p^* \;\text{ and }\; p_B = MC_B(Q) < p^*\,. \tag{9}$$

Incentive compatibility also holds for a maximum price p_A^{max}, except for the A-bidders with higher costs than p_A^{max}, who do not participate. The maximum price is effective if $p_A^{max} < p^*$. Then, by (2) and (5), $p_A = p_A^{max} < p^*$, $p_B > p^*$, $S_A(p_A) < S_A(p^*)$, and $S_B(p_B) > S_A(p^*)$.

With a bonus b, incentive compatibility applies to the A-bidders, whereas the B-bidders reduce their bids by b. The bonus also implies $p_A < p^* < p_B$ and supply volumes $S_A(p_A) < S_A(p^*)$ and $S_B(p_B) > S_B(p^*)$. Incentive compatibility holds for the bid bonus. Since the argumentation is the same as for the monetary bonus, the results also apply to the bid bonus.

Both the maximum price and the bonus imply volume shift (8) as the quota.

To analyze the effect of discriminatory instruments on the support costs, we state three conditions:[1]

(C1) $\varepsilon_A(p)$ and $\varepsilon_B(p)$ are non-increasing in p.

(C2) $S_B(p^*) > 0$.

(C3) $\varepsilon_A(p^*) < \varepsilon_B(p^*)$.

Let $\Delta(q)$ denote the change in the support costs induced by q compared to those in a free competition. Then, (7) and (9) imply

$$\Delta(q) = MC_A(S_A(p^*) - q) \cdot (S_A(p^*) - q) + MC_B(S_B(p^*) + q) \cdot (S_B(p^*) + q) - K(p^*)\,.$$

Differentiating $\Delta(q)$ with respect to q, denoted by $\Delta'(q)$, we obtain

$$\begin{aligned}\Delta'(q) &= -MC'_A(S_A(p^*) - q)(S_A(p^*) - q) - MC_A(S_A(p^*) - q) \\ &\quad + MC'_B(S_B(p^*) + q)(S_B(p^*) + q) + MC_B(S_B(p^*) + q)\,.\end{aligned}$$

We first analyze the effect of discriminatory instruments on the support costs when the instruments become effective. Thus, we consider $\Delta(q)$ at $q = 0$,

$$\Delta'(0) = -MC_A(S_A(p^*)) - S_A(p^*)MC'_A(S_A(p^*)) + MC_B(S_B(p^*)) + S_B(p^*)MC'_B(S_B(p^*))\,.$$

By $MC_A(S_A(p^*)) = MC_B(S_B(p^*)) = p^*$, we obtain

$$\Delta'(0) = S_B(p^*)MC'_B(S_B(p^*)) - S_A(p^*)MC'_A(S_A(p^*)). \tag{10}$$

[1] (C1) is a standard assumption and is supported by the RE literature (**???**). (C2) requires that the B-bidders gain at least a small share in a non-discriminatory auction. There are many examples where wind and solar are awarded in multi-technology auctions, for example, in Mexico (**?**) and Spain (**?**), or are awarded in separate auctions but at similar prices, for example, in Germany (**??**). See also Table **??**. According to (C3), the B-bidders' price elasticity of supply at p^* is larger than that of the A-bidders, which is justified by the B-bidders' smaller supply volume in a non-discriminatory auction.

With $MC_k'(S_i(p)) = \frac{1}{S_k'(p)}$ for $k \in \{L, H\}$,

$$\Delta'(0) < 0 \quad \text{if} \quad \frac{S_A(p^*)}{S_A'(p^*)} > \frac{S_B(p^*)}{S_B'(p^*)} \Longleftrightarrow \frac{S_A'(p^*)}{S_A(p^*)} p^* < \frac{S_B'(p^*)}{S_B(p^*)} p^* \,,$$

which, by (4), holds because of (C3). Therefore, the support costs decrease if the quota Q becomes effective, that is, q becomes positive, the maximum price p_A^{max} becomes effective – that is, $p_A^{max} - p^*$ becomes negative –, or the bonus b becomes positive.

We now show that, given (C1), (C2), and (C3), for each instrument there exists a unique support cost minimizing parameterization and the respective optima are equivalent. The minimization of the support costs

$$K(p_A, p_B) = p_A S_A(p_A) + p_B S_B(p_B) \;\text{ subject to }\; S_A(p_A) + S_B(p_B) = D \tag{11}$$

with regard to p_A and p_B yields the first order conditions

$$\frac{\partial K(p_A, p_B)}{\partial p_k} \quad = \quad S_k(p_k) + p_k S_k'(p_k) + \lambda S_k'(p_k) = 0, \; k \in \{A, B\} \,,$$

which lead to the condition

$$p_B - p_A = \frac{S_A(p_A)}{S_A'(p_A)} - \frac{S_B(p_B)}{S_B'(p_B)} \,. \tag{12}$$

For $Q \leq S_B(p^*)$, $p_B = p_A = p^*$ and, thus, the left-hand side of (12) is zero. $Q > S_B(p^*)$ implies $p_B > p^* > p_A$. As Q increases, p_B increases and p_A decreases and, thus, the left-hand side of (12) increases. (4) together with (C1), (C2), and (C3) imply that the right-hand side of (12) is positive at p^*. Thus, (12) does not hold for an ineffective quota $Q \leq S_B(p^*)$. By (C1), $\varepsilon_B(p_B)$ does not increase if p_B increases and $\varepsilon_A(p_A)$ does not decrease if p_A decreases. Thus, based on (4), the right-hand side of (12) decreases. Since the left-hand side of (12) increases in Q and the right-hand side of (12) decreases, there exists a unique $\hat{Q}$ that fulfills (12). Combined with the property that the support costs decrease when the quota becomes effective, this implies that $\hat{Q}$ is the unique cost minimizing quota. Thus, there exists a unique quota $\hat{Q} > S_B(p^*)$ that minimizes the support costs, where $\hat{Q}$, p_A, and p_B are determined by

$\hat{Q} = S_B(p_B)$, $S_A(p_A) + S_B(p_B) = D$ and

$$p_B - p_A = \frac{S_A(p_A)}{S'_A(p_A)} - \frac{S_B(p_B)}{S'_B(p_B)}\,.$$

Analogously, this also applies to the maximum price and the bonus. Thus, there exists a unique maximum price $\hat{p}_A^{max} > 0$ that minimizes the support costs, where $\hat{p}_A^{max}$, p_A, and p_B are determined by $S_A(\hat{p}_A^{max}) + S_B(p_B) = D$ and

$$p_B - \hat{p}_A^{max} = \frac{S_A(\hat{p}_A^{max})}{S'_A(\hat{p}_A^{max})} - \frac{S_B(p_B)}{S'_B(p_B)}\,,$$

and there exists an unique bonus $\hat{b} > 0$ that minimizes the support costs, where $\hat{b}$ and the award price p are determined by $S_A(p) + S_B(p + \hat{b}) = D$ and

$$\hat{b} = \frac{S_A(p)}{S'_A(p)} - \frac{S_B(p + \hat{b})}{S'_B(p + \hat{b})}\,.$$

From these results, it follows directly that the quota $\hat{Q}$, the maximum price $\hat{p}_A^{max}$, and the bonus $\hat{b}$ lead to the same support-cost-minimizing outcome, that is, the prices (payments) and the supply volumes of the A-bidders and B-bidders are the same for $\hat{Q}$, $\hat{p}_A^{max}$, and $\hat{b}$.