# Discrepancies in scoring auctions for the energy sector

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**Abstract**

Scoring auctions are an appropriate purchasing mechanism if the buyer values the auctioned good in more attributes than just the price. In contrast to ordinary procurement auctions, such auctions facilitate a range of different allocation and payment rules. However, exactly as in ordinary procurement auctions there exist incentive compatible scoring auctions where truthful revelation of costs and quality is an optimal bidding strategy. Such scoring auctions are of special interest in the energy sector. There are different energy markets where electricity generators compete and an independent system operator is eager to know the true generation costs. In this special case, the marginal electricity generation costs are interpreted as quality whereas the stand-by costs of a power plant are the fixed costs in terms of a scoring auction. We formally analyze two different models that apply scoring auctions to the energy sector. We prove that these approaches can under some assumptions lead to the desired results. In general, scoring auctions can be implemented explicitly or implicitly but both result in the same outcome. This result contradicts with the opinion of some authors in that research area who claim that their model is superior to another. We prove this equivalence and give an outlook on the implications. Furthermore, we give a brief prospect on where and how explicit and implicit scoring auctions can also be applied.

*Keywords:* Scoring Auction, Energy, Balancing Power, Multiattributive Auction

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## 1. Introduction

There are many different forms of commercial electricity generation worldwide, from nuclear power plants over coal-fired power plants to photovoltaic power plants. Differences in the functioning and fueling lead to differences on the economic view of these electricity generation plants. Depending on the individual situation, one type of power plant may be more meaningful and economic than another. The economic differentiation is based on different cost types, including fuel costs, operating costs and maintenance costs among others. Hence, there is no one price tag for one unit of electricity (usually in MWh) for one power plant but the overall costs are a function of different parameters.

To dispatch all power generating facilities efficiently, i.e. to use those power plants at each moment in time that cause the least overall costs to produce the required amount of electricity, the responsible system operator has to know all these costs. That is not a major problem in an integrated energy market where system operator and power plants are part of one superior energy company but it is a challenge in all energy market where different energy generation facilities (current or future) compete with each other. This challenge arose through the liberalization of the energy market in some countries which resulted in power plants competing with each other. Thereby, two markets are of special interest, the one for integrating new power plants into an existing grid and the balancing power market.

A powerful market mechanism to cope with this challenge are scoring auctions. The next section will give a compact introduction to this concept before Sections 3.1 and 3.2 will give theoretic examples of an application of this concept to the energy market. After an comparison of both concepts in Section 3.3, Section 4 concludes this work.

## 2. Scoring Auctions

A scoring (procurement) auction can be applied if the auctioned good is characterized by multiple attributes. Besides the price, this attributes are usually referred as quality level of the good and examples are product characteristics (Che, 1993), lead time (Herbsman et al., 1995) or logistics and transportation costs (Kostamis et al., 2009). The following introduction and the notation is based on the work of (Asker and Cantillon, 2008) and (Asker and Cantillon, 2010).

The auctioneer purchases one indivisible unit of a good that is characterized by its price $p$ and its quality level $q \in \mathbb{R}_+^M$ where $M \geq 1$ represent the $M$ different attributes described as quality. The set of bidders participating in the auction is $N = \{1, ..., n\}$ with $n \in \mathbb{N}$. The auctioneer values the quality $q$ of the good according to the valuation function $v : \mathbb{R}_+^M \to \mathbb{R} : (q) \to v(q)$ and thus $\pi_A(p, q) = v(q) - P$ is the auctioneer's profit from acquiring a good with quality $q$ for price $p$.

It is assumed that each bidder $i \in N$ can decide separately which quality $q$ to offer and is furthermore characterized by his type $\theta_i \in \mathbb{R}^K$ with $K \geq 1$. This type generally reflects the bidders' (cost) structure. The resulting costs for supplying a good of quality $q$ given type $\theta_i$ is then given by the cost function $c : \mathbb{R} \times \mathbb{R}^K \to \mathbb{R} : (q, \theta_i) \to c(q, \theta_i)$. The profit of the awarded bidder from selling a good $(p, q)$ given his type $\theta_i$ is then given by $\pi_B(p, q, \theta_i) = p - c(q, \theta_i)$.

Both, $v(q)$ and $c(q, \theta_i)$ are twice continuously differentiable and increasing in $q$[1]. The difference function $w(q, \theta_i) = v(q) - c(q, \theta_i)$ is bounded and concave and represents the social welfare gain of the transaction of a good $(p, q)$ from a bidder with type $\theta_i$. $v(q)$ and $c(q, \theta_i)$ are common knowledge among bidders and auctioneer except bidders type $\theta_i$ for $i \in N$. These types are independently distributed according to the continuous

[1] The auctioneer values a good of higher quality higher and the bidder has higher costs to supply a good of higher quality

density function $f(\cdot)$ and the corresponding cumulative distribution function $F(\cdot)$. $f(\cdot)$ and $F(\cdot)$ are also common knowledge.

The scoring rule $S$ of an auction determines an continuous preference relation to any good $(p, q)$. Hence, $S$ can formally be described as $S : \mathbb{R} \times \mathbb{R}_+^M \to \mathbb{R} : (p, q) \to S(p, q)$. We assume the auctioneer to aim for efficient results and thus the scoring rule is equivalent to the auctioneers profit $S(p, q) = v(q) - p$. As a result, $S$ is quasi-linear as well as increasing and twice continuously differentiable in $q$.

Every bidder $i \in N$ submits either a bid of the type $(b_i^p, b_i^q) \in \mathbb{R} \times \mathbb{R}_+^M$ or equivalently $b_i = S(b_i^p, b_i^q)$. The bidder with the highest score is awarded in the auction [2]. In contrast to a price only auction, there are more varieties to the standard first- and second-price auctions. A first-price auction could imply that an awarded bidder $i$ has to supply a good $(p, q)$ with $(p, q) = (b_i^p, b_i^q)$ but it is also possible that he has only to fulfill the condition $S(p, q) = b_i$. There are even more possibilities when considering second-price auctions. For the remainder of this paper we will speak of first- and second-score auctions which means that the awarded bidder has to supply a good $(p, q)$ with $S(p, q) = S^w = S(p^w, q^w)$ where $(p^w, q^w)$ is either in case of a first-score auction the bid of the awarded bidder or in case of the second-score auction the bid of the best not awarded bidder.[3]

Given those conditions, an awarded bidder $i$ with type $\theta_i$ seeks to maximize his profit $\pi_B(p, q, \theta_i$ with the restriction that the supplied good $(p, q)$ satisfies the requirement $S(p, q) = S^w$. Independent of the pricing rule, the maximization problem in case of winning is thus

[2] In case of a multi-unit auctions with a $k$ good demand, the bidders with the $k$ highest scores are awarded.

[3] The analysis regarding the equilibrium will show that this restriction does not affect the bidding behavior for most variations of this pricing rules.

$$\max_{(p,q)}\{p - c(q,\theta_i)\} \tag{1}$$

$$\text{s.t. } v(q) - p = S^w. \tag{2}$$

Solving (2) for $p$ and substituting it into (1) leads to

$$\max_{q}\{v(q) - c(q,\theta_i) - S^w\}. \tag{3}$$

Hence, for every bidder $i \in N$ there is a quality $q_i^*$ which maximizes (3) which is independent of $S^w$ and thus also maximizes bidders profit. As an implication, a bidder will always supply a good of quality $q_i^*$ independent of the score he has to fulfill. Moreover, this quality also maximizes the social welfare $w(q,\theta_i)$. The maximum social welfare a specific bidder $i$ can generate $w(q_i^*,\theta_i)$ is, given the valuation and cost function, solely determined by the bidder's type $\theta_i$ and in the following this is called the bidder's pseudotype $\lambda(\theta_i)$ with

$$\lambda(\theta_i) = \max_{q}\{v(q) - c(q,\theta_i)\}. \tag{4}$$

The higher the pseudotype $\lambda(\theta_i)$ of a bidder, the lower are the costs of this bidder to fulfill any Score $S$. Furthermore, there is no incentive for a bidder to deviate from his specific quality $q_i^*$ as an increase in the quality would lead to a higher rise of costs than of valuation. The pseudotype $\lambda(\theta_i)$ of any bidder $i\ inN$ is defined as soon as valuation and cost functions are given. Thus, from the distribution function $F(\cdot)$ of $\Theta$ the distribution function $G(\cdot)$ with corresponding density function $g(\cdot)$ of $\Lambda(\Theta)$ can be derived. Bidder $i$'s expected profit can be calculated as

$$E_S[\pi_i(b_i^p, b_i^q, \theta_i] = (\lambda(\theta_i) - S^w)P(S(b_i^p, b_i^q) > \max_{j \in N, i \neq j} S(b_j^p, b_j^q)) \quad (5)$$

which is the same problem as in a standard IPV auction and thus the same solution concepts apply. An important implication of this finding is that a second-score auction is incentive compatible, both in the price as well as in the quality component.

## 3. Application of Scoring Auctions in the Energy Sector

In the energy sector and more particular in the field of electricity generation scoring auctions can be applied. In general, different electricity generation plants at different locations, with different technologies and with different preconditions supply electricity for a given market (Wilson, 2002). These variety of generators is caused by the special characteristics of the electricity market. These special characteristics are explained simplified hereafter.

A general principle of the electricity market is that supply and demand have to be balanced at any moment in time. The balancing is the assignment of the so called system operator. The system operator can not influence the electricity demand, hence, he has to dispatch the electricity generation to balance the system. The fluctuation in demand and the need for dispatching the plants leads to fluctuating market prices for electricity. Those fluctuations can be represented by two curves, the load-duration curve and the cost-duration curve. The load duration curve $l(t) \in [0, 1]$ represents the cumulative electricity demand distribution over time. It assigns to every load level $l \in \mathbb{R}_+$ the percentage of a given time period $t(l)$ where at least this load level is reached. Hence, there result the following properties for the load-duration curve: $l \to \infty : t(l) \to 0$ and $l \to 0 : t(l) \to 1$.

The cost-duration curve $\gamma(p^c) \in [0, 1]$ connects the load-duration curve $l(t)$ with the merit order of a given electricity market.[4] The result is a curve that assigns to every price $p^c \mathbb{R}_+$ the percentage of a given time period $\gamma(p^c)$ where the price for electricity is at least as high as $p^c$. These curve contains valuable information for potential electricity generators as they can estimate potential revenues from selling electricity at the market depending on their marginal costs of providing electricity.

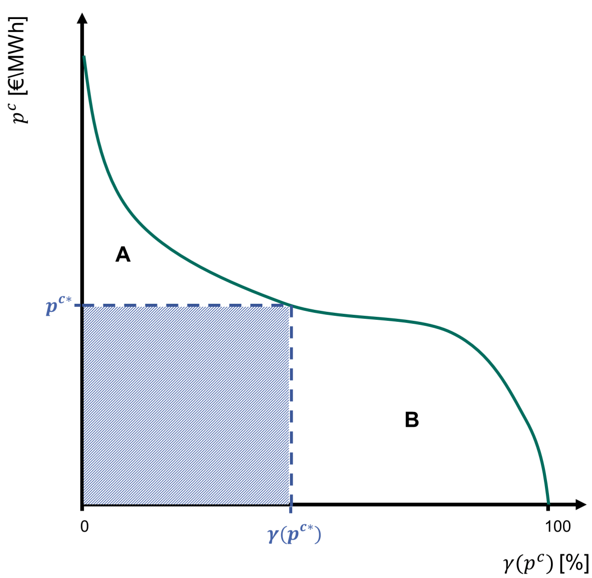


Figure 1: Example of a cost-duration curve.

As a result of this market structure, there is a need for different types of electricity generation plants with different characteristics. Broadly speaking, every power plant $i$ of all power plants $N = \{1, ..., n\}$ can economically be described by two cost components. A fixed component $k_i \in \mathbb{R}_+$ that represents the costs to hold the plant available, i.e. costs for maintenance, permanent personal, etc., and is independent of the actual electricity generation. The other cost component $c_i \in \mathbb{R}_+$ represents the variable costs of producing electricity. Those costs are also referred to as marginal costs and include

[4]The merit order is the increasing order of the marginal costs of all power plants in the market with their respective capacity.

fuel costs and costs of operation. For the remainder of this paper, we will assume that $c_i$ and $k_i$ are independently distributed according to the density functions $f(\cdot)$ and $g(\cdot)$ and the respective cumulative distribution functions $F(\cdot)$ and $G(\cdot)$ respectively.

### *3.1. Concept of Bushnell and Oren (1994)*

Bushnell and Oren (1994) apply the concepts of scoring auctions to a field of the electricity market that emerged from the deregulation in the energy sector, in particular private power plants compete with each other in an electricity market. In case a new plant is needed, the main question is which plant $i$ offers the best combination of $k_i$ and $c_i$ to the system operator. Thereby it is crucial for the system operator to know the true marginal electricity generation costs $c_i$ to efficiently dispatch all plants within the market. The authors make the following notable assumptions to the model. First, the cost-duration curve $\gamma(p^c)$ is common knowledge among the competitors. Second, each new plant would only add marginal capacity to the whole market. And third, each plant receives for the whole time it is called, i.e. it produces electricity for the market, the same price $p^c$ for the delivered electricity. This section first derives scoring and payment rule to the general principles of a scoring auction and then compares them to the results of Bushnell and Oren (1994). At the end of the section, the pseudotype and bidding behavior are derived in accordance with the approach of Asker and Cantillon (2008).

#### *Scoring and Payment Rule*

For the winning bidder $i$ this yields to a payoff of $\pi_i(p^k, p^c, k_i, c_i) = (p^k - k_i) + (p^c - c_i)\gamma(p^c)$, where $p^k$ is the compensation the winning bidder receives for the fixed costs $k_i$. An implication of this design is that the bids are of the following form for every bidder $i \in N$: $(b_i^k, b_i^c)$, where $b_i^k$ represents the fixed bid component and $b_i^c$ the variable one. The determination of the buyer's profit is much harder. First of all, it is obvious that it is linear in the fixed payment $p^k$ and thus the variable payment per energy unit $p^c$ is viewed as quality in this model due to the following reasoning. The auctioneer, i.e. the system operator, has to pay a fixed price to obtain the opportunity to receive electricity at a specific rate from the bidder but also to curtail the power generation of this specific bidder. A specific price change in the fixed payment always leads to the same change in profit independent of the base value. That does not hold for the variable payment. As a result the following formula can be interpreted as the buyers payoff: $\pi_A(p^k, p^c) = v(p^c) - p^k$.

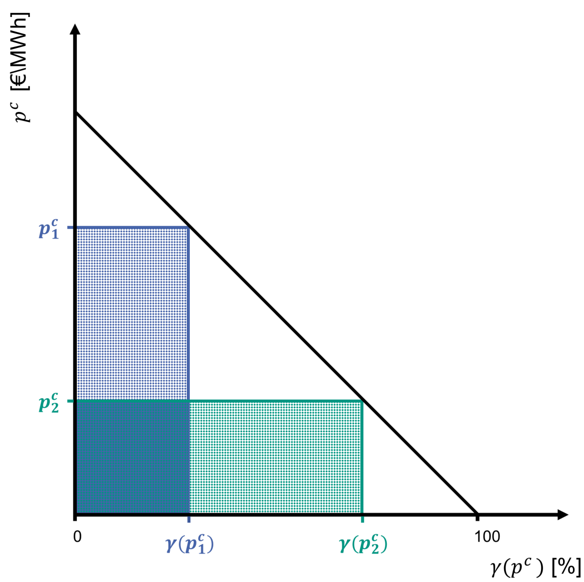


Figure 2: Simplified example of a cost-duration curve with two different prices.

The remaining question is, what is the valuation? A natural approach would be to measure the costs a specific variable payment per energy unit $p^c$ induces. It is obvious that a lower price should have a higher valuations than a higher price. So the pure payment the bidder receives for the electricity generation is not the correct choice. This is illustrated in the simplified example of Figure 2. In this example there are two players, 1 and 2, with two prices $p_1^c$ and $p_2^c$ respectively. It can be clearly seen that $p_2^c$ is lower than $p_1^c$ and should be higher valued by the auctioneer. Hence, if one compares the payments both players would receive, which is highlighted by the correspond shaded areas, one can recognize that it is the same for each player. Player 1 supplies less energy but for a higher price and for player 2 it is vice versa. There could even be a cost-duration curve where every potential price would result in the same payment.

The right choice is to take the advantage the auctioneer has from a participating bidder over the status quo into account. Therefore, it is not important to know something about costs of the winning bidder but just what price he receives for supplying electricity. In Figure 1 this price is denoted by $p^{c*}$ and the corresponding revenue is the shaded area. This payment is not crucial as the buyer would have to pay this price for the energy anyway due to the fact that the participation of an additional bidder changes the cost-duration curve just infinitesimal (see assumption above). But since there is a supplier who potentially generates energy for the price $p^{c*}$, all suppliers with higher costs are now more often curtailed. The difference for each supplier individually is infinitesimal, but for the buyer this means savings for every hour with a price above $p(\gamma) = p^{c*}$, where $p(\gamma)$ is the inverse function of $\gamma(p^c)$. These savings are illustrated in Figure 1 as the white area $A$ between price $p^{c*}$ and cost-duration curve. As a result the valuation for a specific variable price $p^c$ for the buyer is as follows

$$v(p^c) = \int_0^{\gamma(p^c)} (p(\gamma) - p^c) d\gamma. \tag{6}$$

As we assume that the system operator seeks for an efficient market, the scoring rule is then defined as

$$S(b^k, b^c) = \int_0^{\gamma(b^c)} (p(\gamma) - b^c) d\gamma \ - \ b^k. \tag{7}$$

For the payment rule, it is from utmost importance that it leads to incentive compatible bidding behavior, especially regarding the variable cost component. In Section 2 we showed that a second-score payment rule leads to the desired behavior. So how does a second-score payment rule look like in this case? We consider a winning bidder $i$ and the first losing bidder $j$ and their bids $b_i = (b_i^k, b_i^c)$ and $b_j = (b_j^k, b_j^c)$ respectively. The scoring function $S(b^k, b^c)$ maps these bids onto a score $s_i = S(b_i^k, b_i^c)$ and $s_j = S(b_j^k, b_j^c)$. As we know that $i$ won the auction it immediately follows that $s_i > s_j$ but there is no statement regarding the relationship of the fixed or variable components of the score of these two bidders possible. As already mentioned, the marginal electricity costs are regarded as quality and thus, there is the intention to pay the winning bidder the variable costs $p^c = b_i^c$. As a result, the second-price characteristic can only be implemented in the fixed cost compensation payment $p^k$. Hence, this condition combined with the idea of a second-score payment rule implies that the winning bidder receives a compensation payment $p^k$, while retaining her quality bid $b^c$ as variable payment, that would gain the same score as the second highest bidder bid $s_j = S(p^k, b_i^c)$. Defining the inverse function of the scoring rule as

$$S^{-1}(s, b^c) = \min \left\{ b^k | S(b^k, b^c) = s \right\}$$

leads to the following payment rule for the winning bidder $i$ given that $j$ is the first losing bidder with $S(b_j^k, b_j^c) = s_j$:

$$p^c = b_i^c \tag{8}$$

$$p^k = S^{-1}(s_j, b_i^c). \tag{9}$$

Given scoring rule (7), the inverse function and thus payment rule for the fixed compensation payment is

$$p^k = b_j^k + \int_{b_i^c}^{b_j^c} \gamma(p^c) dp^c. \quad (10)$$

Thus the winning bidder $i$ receives the fixed bid of the first losing bidder $j$ plus the additional savings her variable bid generates for the buyer in comparison to that of the first losing bidder.

*Comparison with results of Bushnell and Oren (1994)*

We derived both, the scoring rule and the payment rule in this section, by applying the principles of scoring rules from Section 2. We will now compare those rules with the findings of Bushnell and Oren (1994), which claim to lead to truthful revelation of the marginal electricity costs. Therefore, they deploy three propositions that lead to two main conditions. Firstly, the payment rule has to look exactly like described in (8) and (9) [5]. Secondly, the scoring rule must be of the following form

$$S(b^k, b^c) = V(b^k + \int_o^{b^c} \gamma(p^c) dp^c), \quad (11)$$

where $V(\cdot)$ is a strictly decreasing function. The outcome is then invariant to the specific form of $V(\cdot)$.[6] In the following, we prove that scoring rule (7) satisfies this condition.

*Proof that scoring rule* (7) *satisfies* (11). First we consider the entire area underneath the cost-duration curve in Figure 1 as a constant. This area, called $C$ is given by

$$C = \int_0^{p_{MAX}^c} \gamma(p^c) dp^c,$$

[5]Proposition 2 in (Bushnell and Oren, 1994).

[6]Proposition 3 in Bushnell and Oren (1994) where $V(\cdot)$ is denoted as strictly increasing function because in their model the bidder with the lowest score wins the auction unlike the model described here, where the highest score wins. Therefore, the scoring rule has a reversed sign.

where $p^c_{MAX} < \infty$ is the highest possible price and $\gamma(p^c_{MAX}) = 0$. $C$ has the same value for every bidder and every potential marginal electricity cost bid. The specific realization of $V(\cdot)$ that leads from (11) to (7) is $V(x) = -x + C$ which is a strictly decreasing function in $x$.[7]

$$
\begin{aligned}
S(b^k, b^c) &= -(b^k + \int_o^{b^c} \gamma(p^c)dp^c) + C \\
&= -b^k - \int_o^{b^c} \gamma(p^c)dp^c + \int_0^{p^c_{MAX}} \gamma(p^c)dp^c \\
&= -b^k + \int_{b^c}^{p^c_{MAX}} \gamma(p^c)dp^c \\
&= -b^k - \int_{\gamma(b^c)}^{\gamma(p^c_{MAX})} p(\gamma)d\gamma + p^c_{MAX}\gamma(p^c_{MAX}) - b^c\gamma(b^c) \\
&= -b^k + \int_0^{\gamma(b^c)} p(\gamma)d\gamma - b^c\gamma(b^c) \\
&= \int_0^{\gamma(b^c)} (p(\gamma) - b^c)d\gamma - b^k.
\end{aligned}
$$

□

*Pseudotypes*

The last paragraph proved that scoring and payment rule correspond to the results of Bushnell and Oren (1994), we will now show that it also holds given the general maximization calculus in a second-score auction. Furthermore, we will derive the corresponding pseudotype for this model. If the variable energy price is interpreted as quality

[7] Laisant (1905) proves that the fourth equation holds.
The intuition behind this proof is as follows. In Figure 1, the scoring rule (7) suggests to add the area which is denoted $A$ to the score. However (11) implies to subtract the shaded rectangle and the area on the right hand side, which is denoted $B$, from the score. Comparing the areas ($A$ compared to $B$ plus the shaded area), both have the same value for each possible bid $b^c$ up to a positive constant. This constant is the area underneath the curve $\gamma(p^c)$, which is here denoted $C$. It can proved that this area is the same for each bidder and each potential bid and the result of a summation of the entire area $C$, $B$ and the shaded area is area $A$.

in this scoring auction, the cost function for a bidder $i$ with fixed cost component $k_i$ and variable cost component $c_i$ to deliver a good of quality $b_i^c$ is $c(b_i^c, k_i, c_i) = k_i - (b_i^c - c_i)\gamma(b_i^c)$ and thus the optimization problem the winning bidder faces is as follows

$$\max_{(b_i^k, b_i^c)} \left\{ b_i^k - c(b_i^c, k_i, c_i) \right\}$$
$$\text{s.t. } v(b_i^c) - b_i^k = s_j,$$

where $s_j$ corresponds to the score the winning bidder has to satisfy. Substituting $b_i^k$ into the maximization problem leads to

$$\max_{(b_i^c)} \left\{ v(b_i^c) - s_j + (b_i^c - c_i)\gamma(b_i^c) - k_i \right\}.$$

As $s_j$ is a constant from bidders' perspective, the value of $b_i^c$ that maximizes the payoff is independent of $s_j$ and hence the pseudotype $\lambda(k_i, c_i)$ of bidder $i$ is

$$\lambda(k_i, c_i) = \max_{(b_i^c)} \left\{ v(b_i^c) + (b_i^c - c_i)\gamma(b_i^c) - k_i \right\}, \tag{12}$$

which is equal to the maximum social surplus bidder $i$ could generate. So the same interpretation as in Section 2 holds. We now prove that $b_i^c = c_i$ maximizes (12) and thus the payoff of bidder $i$.

*Proof that $b_i^c = c_i$ maximizes* (12). To find the derivation of the maximization problem of (12) the derivation of the valuation function $v(b_i^c) = \int_0^{\gamma(b_i^c)} p(\gamma) - b_i^c d\gamma$ is required

$$\frac{\partial v}{\partial b_i^c} = -\gamma(b_i^c).$$

This result leads to the following first-order condition

$$
\begin{aligned}
\frac{\partial}{\partial b_i^c} &= \frac{\partial v}{\partial b_i^c} + \gamma(b_i^c) + (b_i^c - c_i)\frac{\partial \gamma}{\partial b_i^c} \\
&= -\gamma(b_i^c) + \gamma(b_i^c) + (b_i^c - c_i)\frac{\partial \gamma}{\partial b_i^c} \\
&= (b_i^c - c_i)\frac{\partial \gamma}{\partial b_i^c} \overset{!}{=} 0.
\end{aligned}
$$

As the derivation of $\gamma(b_i^c)$ is unequal to zero for all feasible $b_i^c$ the unique solution is $b_i^c = c_i$.

For the sake of completeness the result of the second-order condition is

$$
\begin{aligned}
\frac{\partial^2}{\partial^2 b_i^c}(c_i) &= \frac{\partial \gamma}{\partial b_i^c}(c_i) + (b_i^c - c_i)\frac{\partial^2 \gamma}{\partial^2 b_i^c}(c_i) \\
&= \frac{\partial \gamma}{\partial b_i^c}(c_i) < 0
\end{aligned}
$$

and hence satisfies the requirements of a (local) maximum. □

The resulting pseudotype

$$
\lambda(k_i, c_i) = v(c_i) - k_i \tag{13}
$$

is exactly the score of the true costs. In a second-score auction the optimal bidding strategy is consequently $b_i = (b_i^k, b_i^c) = (k_i, c_i)$, that is truth-revealing.

### *3.2. Concept of Chao and Wilson (2002)*

The main critique of the model of Bushnell and Oren (1994) is the assumption regarding the cost-duration curve $\gamma(p^c)$. It is assumed that neither a bidder can have influence on this curve nor does it change over time. Furthermore, all bidders believe that $\gamma(p^c)$ is correct. In a dynamic and long-term market like in the energy sector, these assumptions are hard to maintain. This is in particular true for the balancing power market. This market, which is also part of the energy sector, contrasts with the market for new electricity generation plants that was discussed in the previous section as it does not consider only one additional unit of infinitesimal capacity but the whole market capacity will be tendered (Wilson, 2002). Obviously, the assumptions regarding $\gamma(p^c)$ do not hold any more in this case. In this section, we will describe the model of Chao and Wilson (2002) that avoids, on its own admission, the problems regarding $\gamma(p^c)$ and subsequently analyze it regarding bidding behavior to check whether it keeps its promise.

Although both markets are different, in the formal description there are many similarities. First, we will simplify the market in that sense that we assume all bidders to bid the same capacity that is, compared to the overall market volume, too small to influence the market price. Furthermore, we will only consider the market for positive balancing power, the reasoning for the negative market is analogous. Secondly, the bidders in this market can again economically be described by two cost components $k_i$ and $c_i$ although the interpretation is different. $k_i$ are the stand-by costs of a power plant. It can not participate electricity wholesale market and has to maintain readiness of the plant. $c_i$ are again the costs of actually running the plant. From an auctioneer's perspective it is again of utmost interest to know the true variable costs $c_i$ of the bidders to determine the efficient order of the generation calls.

The demand for balancing power fluctuates during a given time period as demand forecast and the actual demand coincide more or less during this time span. Thus, there are different market prices depending on the balancing power demand. The price, the last called bidder receives for an additional unit of reserve energy in the ascending order of costs, is denoted as spot market price $p^*$. The distribution of the spot market price over the time period that is relevant for the next auction can be concluded through experiences with foregone auctions and forecasts. This distribution function is denoted $\gamma(p^c)$ and has the same properties as the identically named cost-duration curve in Section 3.1. As there are several bidders awarded in the auction and the spot price fluctuates during each time period, there is no simple second-price auction possible. In such a more complex environment, a uniform-price auction (Cramton, 1998) is held. This means each awarded bidder is called to supply balancing power whenever the spot price $p^*$[8] is higher or equal to the energy bid $b_i^c$ of this particular bidder $i$. This supplier is paid the latest spot price for each energy unit he supplies. Note, this all holds just for bidders which were awarded in the auction that was held before they were called for balancing power supply. The part of the profit generated from actual energy supply can then be formulated as:

$$\pi_i^c(b_i^c, c_i) = \gamma(b_i^c)E[p^* - c_i \mid p^* \leq b_i^c]. \tag{14}$$

That is, the probability $\gamma(b_i^c)$ that the spot market price is higher or equal than the bid variable energy price $b_i^c$ times the expected value of the difference between spot price $p^*$ and actual costs $c_i$, given that the spot market price is higher or equal to the bid.

Figure 3 visualizes this profit in a simplified example. In this example the variable costs $c_i$ are lower than the price bid $b_i^c$ and hence, the profit is the sum of the two shaded areas (green and blue). The boundaries of this area are the probability function $\gamma(p^*)$, the marginal costs $c_i$ and the probability that the spot price is higher or equal to the price bid $\gamma(b_i^c)$. The interpretation behind this area is that every price $p$, higher or equal to the bid (thus left of $\gamma(b_i^c)$), has the probability $\gamma(p)$ and generates a profit

[8] It is assumed that the probability distribution over the possible spot prices is continuous and hence the probability that the spot price has a specific value $p$ is zero $P(p^* = p) = 0$.

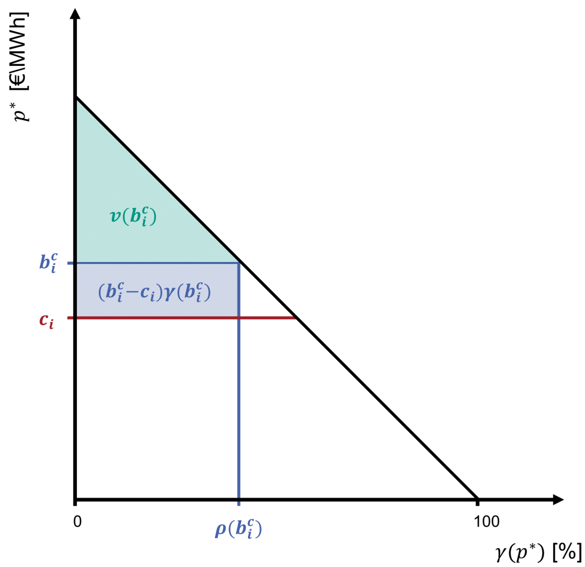


Figure 3: Simplified visualization of bidder $i$'s profit.

of $p - c_i$. To learn whether it is an optimal bidding strategy for bidder $i$ to truthfully reveal his variable costs $c_i$ we now consider the profit from supplying balancing power in case of being awarded in more detail.[9]

$$\begin{aligned}
\pi_i^c(b_i^c, c_i) = \gamma(b_i^c)E[p^* - c \mid p^* \leq b_i^c] \\
&= \quad \gamma(b_i^c)(E[p^* \mid p^* \leq b_i^c] - c_i) \\
&= \quad \gamma(b_i^c)\frac{\int_0^{\gamma(b_i^c)} p(\gamma)d\gamma \ - c_i}{\gamma(b_i^c)} \\
&= \quad \gamma(b_i^c)\frac{\int_0^{\gamma(b_i^c)} p(\gamma) - c_i \ d\gamma}{\gamma(b_i^c)} \\
&= \quad \int_0^{\gamma(b_i^c)} p(\gamma) - c_i \ d\gamma.
\end{aligned}$$

[9]The last equality holds because the rectangle $c_i \cdot \gamma(b_i^c)$ can also be expressed as $\int_0^{\gamma(b_i^c)} c \ d\gamma$.

The optimal bidding strategy can now be derived easily. The value of the integral $\int_0^{\gamma(b_i^c)} p(\gamma) - c_i \; d\gamma$ increases up to the point when $b_i^c = c_i$ and decreases from there on. This can also be seen in Figure 3 where at $b_i^c = c_i$ the profit increases by the white triangle between blue area and curve $\gamma(p^*)$. Hence, the profit an awarded bidder receives from supplying balancing power and bidding $b_i^c = c_i$ is

$$\pi_i^c(c_i, c_i) = \int_0^{\gamma(c_i)} p(\gamma) - c_i \; d\gamma, \tag{15}$$

which is essentially the same as the valuation function of the buyer in Section 3.1. The only difference is, that in this case it is the producer surplus and in the other case it is the consumer surplus. The payment rule for the fixed cost component is also uniform pricing, i.e. every awarded bidder receives the fixed price the bidder with the highest rejected score bid. In the following this price is denoted by $p^k$ and the overall profit of an awarded bidder $i$ is $\pi_i(b_i^k, b_i^c, k_i, c_i) = \pi_i^c(b_i^c, c_i) + p^k - k_i$. Thus, the winning bidders profit is independent of their respective fixed price bid $b_i^k$. However, the probability of being awarded is depending on this bid component.

Before considering the scoring rule, we analyze the profit of the auctioneer. As he always pays the spot market price for the full amount of balancing power demanded at every moment in time, the profit from actually receiving balancing power is zero. Additionally, the auctioneer has a negative profit from buying potential capacity in $\pi_A(p^k, p^c) = -p^k$ for each bidder. He may have also a valuation for this capacity available but this is not considered in this paper, neither in this section nor in the previous one, as it is not depending on the actual price and thus only a constant. Once more, we assume the auctioneer seeks efficient outcomes and thus the scoring rule represents the auctioneer's profit

$$S(b_i^k, b_i^c) = -b_i^k \tag{16}$$

This means, the bidders with the lowest fixed price bids are awarded in the auction.[10] It may seem to be a price only auction but firstly, the bidders are required to also submit their variable price bid to calculate the dispatch order, secondly, the variable costs have also an influence on the fixed price bid as both costs and bids are relevant for the profit.

We already showed that it is optimal for any bidder $i \in N$ to bid $b_i^c = c_i$, the next step is to analyze the optimal bidding strategy for the fixed price bid. In an uniform price auction where the highest rejected bid determines the price and single unit supply, it is a weakly dominant strategy to bid that price that would lead to a profit of zero if it would be the award price (Weber, 1983). Hence, the optimal bid for the fixed cost component is

$$b_i^k = \pi_i^c(c_i, c_i) - k_i. \tag{17}$$

Thus, the bidder truthfully reveals his variable costs $c_i$ and prices the resulting profit from supplying balancing power in his fixed price bid in. And thus, the fixed price bid equals by definition the pseudotype of bidder $i$

$$\lambda_i(k_i, c_i) = \pi_i^c(c_i, c_i) - k_i. \tag{18}$$

As a result, there is an efficient outcome and the main goals are achieved. There is no need for the auctioneer the publish an official estimation or forecast of the function $\gamma(p^*)$ and thus it is not important whether the bidders believe this forecast or not which is a major downside of the model of Bushnell and Oren (1994). If this approach also gains an advantage regarding uncertainties in the forecast and development of $\gamma(p^*)$ and if there are main differences regarding the bidding incentives will be analyzed in

[10] The negation is necessary to be consistent throughout the paper where always the highest score wins.

the next seccction.

*3.3. Comparison of both Concepts*

This section will compare the models of Bushnell and Oren (1994) and Chao and Wilson (2002) regarding the profit of bidders and auctioneer, characteristics of the strategies and general requirements and assumptions. To make both models comparable we will only consider one winning bidder.

At first we compare the pseudotype of the bidders in the two auction models. During the remainder of this section we will always denote variables from the model of Bushnell and Oren (1994) by a superscript $^{B/O}$ and those of the model of Chao and Wilson (2002) with $^{C/W}$. The respective pseudotypes are given in (13) and (18).

$$\begin{aligned}
\lambda_i^{B/O} &= v(c_i) - k_i \\
&= \int_0^{\gamma(c_i)} (p(\gamma) - c_i) d\gamma - k_i \\
&= \pi_i^{c,C/W}(c_i, c_i) - k_i \\
&= \lambda_i^{C/W}.
\end{aligned}$$

So given the costs $k_i$ and $c_i$, payment and scoring rule in both models, the pseudotypes are alike. Recall that the pseudotype corresponds the type in a classical IPV auction. Next, we analyze differences in the profit of bidders and auctioneer. First the profit of a winning bidder under the assumptions that all bidders bid the strategy we derived in the respective section of this paper

$$\begin{aligned}\pi_i^{B/O}(b_i^k, b_i^c, k_i, c_i) &= (p^{k,B/O} - k_i) + (b_i^c - c_i)\gamma(b_i^c) \\ &\overset{b_i^c = c_i}{=} (p^{k,B/O} - k_i).\end{aligned}$$

The fixed payment $p^{k,B/O}$ is calculated as follows

$$\begin{aligned}p^{k,B/O} = S^{-1}(S(b_j^k, b_j^c), b_i^c) &= b_j^k + \int_{b_i^c}^{b_j^c} \gamma(p^c)dp^c \\ &= b_j^k + v(b_i^c) - v(b_j^c) \\ &= k_j + v(c_i) - v(c_j).\end{aligned}$$

Overall this leads to

$$\pi_i^{B/O}(b_i^k, b_i^c, k_i, c_i) = (k_j - k_i) + (v(c_i) - v(c_j)), \tag{19}$$

which is equal to the additional welfare bidder $i$ can generate over bidder $j$. In the model discussed in this section the winning bidder has the profit

$$\begin{aligned}\pi_i^{C/W}(b_i^k, b_i^c, k_i, c_i) &= \int_0^{\gamma(b_i^c)} p(\gamma) - c_i \; d\gamma + (p^{k,C/W} - k_i) \\ &\overset{b_i^c = c_i}{=} v(c_i) + (p^{k,C/W} - k_i),\end{aligned}$$

where $v(\cdot)$ is the valuation introduced in (19). The fixed payment is in this case

$$
\begin{aligned}
p^{k,C/W} = S^{-1}(S(b_j^k, b_j^c), b_i^c) &= -b_j^k \\
&= -\left(\int_0^{\gamma(c_j)} p(\gamma) - c_j \; d\gamma - k_j\right) \\
&= k_j - v(c_j).
\end{aligned}
$$

This results in

$$
\pi_i^{C/W}(b_i^k, b_i^c, k_i, c_i) = (k_j - k_i) + (v(c_i) - v(c_j))\,. \tag{20}
$$

Thus, as (19) equals (20), the profit for the bidders are the same under the given assumptions in both models. The profit of all bidders that are not awarded in the auction is zero. This hints that also the auctioneer's profit is the same in both models. Given the calculated prices, the auctioneer's profit in the model Bushnell and Oren (1994) is

$$
\begin{aligned}
\pi_B^{B/O}(p^{k,B/O}, p^{c,B/O}) &= v(p^{c,B/O}) - p^{k,B/O} \\
&= v(c_i) - (k_j + v(c_i) - v(c_j)) \\
&= v(c_j) - k_j.
\end{aligned}
$$

This is exactly the profit the buyer would gain from the cost structure of the first losing bidder $j$. This emphasizes the second-price characteristics used to construct the scoring and payment rule. And in the model of Chao and Wilson (2002) there is the same result

$$\pi_B^{C/W}(p^{k,C/W}, p^{c,C/W}) = -p^{k,C/W}$$
$$= v(c_j) - k_j.$$

These results are summarized in Proposition 1.

**Proposition 1** (Profit equivalencence in the models of Bushnell and Oren (1994) and Chao and Wilson (2002))**.** *Under the assumption that the cost-duration function $\gamma(p^c)$ is common knowledge among the bidders, is equivalent to the belief of the bidders and neither changes during nor after the auction, the following holds if the bidders play the symmetric equilibrium strategies:*

(1) *The pseudotypes of a bidder with the same costs $k_i$ and $c_i$ is the same in both models.*

(2) *The profit of all bidders is the same in both models.*

(3) *The profit of the buyer is the same in both models.*

## 4. Conclusion

So, both scoring auctions, the explicit one from Bushnell and Oren (1994) as well as the implicit[11] one from Chao and Wilson (2002) lead to the same main results. The bidders have the same incentives and have to have the same information. As already mentioned, both models differ in this regard that the auctioneer does not have to publish a cost-duration curve in the model of Chao and Wilson (2002) and thus, there is no risk that some bidders have a different belief. However, both models are equally affected by uncertainties and changes in the cost-duration function.

[11]Implicit because the scoring rule corresponds a price only auction.

A formalized look at the strategic considerations of the bidders reveal that they are almost the same independent if it is an explicit scoring auction (Bushnell and Oren, 1994) or an implicit one (Chao and Wilson, 2002). Therefore, from a formal point of view, both principles lead to the same results although we recognize that there might be differences in practical application. Nevertheless, the advantages that Chao and Wilson (2002) claim for their model can not be confirmed, instead this paper refutes this argument.

Scoring auctions are also applied in other sectors, for example in the construction sector. Herbsman et al. (1995) analyze explicit and implicit scoring auctions in highway construction projects. For such projects, it can also be shown that implicit and explicit scoring auctions lead to the same results. The reasoning is the same. Bidders participating in implicit scoring auctions price their additional payoff due to their quality in their bids in.

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