

# Multi-unit Auctions with Non-binding Award and Reallocation – Theoretical and Experimental Analysis

Matej Belica<sup>a</sup>, Karl-Martin Ehrhart<sup>a</sup>, Marie-Christin Haufe<sup>a</sup>

<sup>a</sup>*Karlsruhe Institute of Technology (KIT), Germany*

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*Keywords:* experimental economics, multi-unit auction, non-binding award, pay-as-bid auction, renewable energy, uniform price auction

*JEL:* D01, D02, D04, D44, Q21

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## **Extended Abstract**

Common auction theory literature concentrates on auctions in which bids and awards are binding. In real-world applications, however, there are examples of auctions in which bidders are allowed to withdraw their bids during the auction or even to reject their award after the auction. The main reason for a non-binding award is to reduce bidders' risks after being awarded, e.g. financing risks. We develop a theoretical model in order to analyze non-binding awards in sealed-bid multi-unit auctions. We report that non-binding awards in combination with reallocation do not have unfavorable effects in Pay-as-bid, Uniform Price and Vickrey Auctions, i.e. efficiency and revenue equivalence still apply. Our experimental study supports our theoretical results.

One example of an auction with non-binding awards is the Australian license auction for satellite-television services in 1993, where two licenses were put up to sale and the bidders were allowed to submit as many bids as they wanted with zero withdrawal costs. Consequently, the winning bidders kept on withdrawing their award just to regain it with a lower bid after reallocation. This happened for 19 times, after which the licenses were allocated to the very same bidders but with a delay of one year and at approximately half of the initial awarding price.<sup>1</sup> In 2009, the Centers for Medicare and Medicaid Services (CMS) conducted auctions

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*Email addresses:* [belica@kit.edu](mailto:belica@kit.edu) (Matej Belica), [ehrhart@kit.edu](mailto:ehrhart@kit.edu) (Karl-Martin Ehrhart), [mariechristin.haufe@gmail.com](mailto:mariechristin.haufe@gmail.com) (Marie-Christin Haufe)

<sup>1</sup>For more details see McMillan (1994).

to replace administrative pricing of medical services allowing non-binding bids with minor success: the auction neither satisfied demand nor generated competitive prices. Cramton, Ellermeyer, and Katzman (2015) theoretically and Merlob, Plott, and Zhang (2012) experimentally point to the major problem in the design of the CMS auction: the combination of the median-pricing rule and the option to withdraw bids after the price has been announced.

Even more recently, the revised German Renewable Energies Act (EEG) establishes that the financial support for new ground-mounted photovoltaic (PV) plants in Germany is determined by a competitive auction mechanism from April 2015 onwards. With this amendment, the government aims to control the amount of new installations and to support those projects that demand the lowest support levels. However, auctions entail award risks for project developers, that is, they face high planning uncertainties before their projects are awarded in the auction. Thus, they may realize after the auction that the awarded support levels are insufficient for getting further agreements for funds, e.g., and for finishing their projects. In order to reduce these risks, winning bids will be non-binding. On that account, the mechanism regulates that project developers who are successful in the auction have to deposit financial securities within ten working days after the auction to ensure their intent to implement the plant in time. Otherwise, i.e. if a bidder does not deposit the required security, her award expires and a reallocation procedure starts: the best losing bids will move up and be awarded instead.<sup>2</sup> Thus, since the award is not binding, bidders have additional strategic options, which may induce further incentives, e.g. the submission of multiple bids for one ground-mounted PV plant.

We develop an independent private values model for multi-unit procurement auctions with homogeneous goods and single-unit supply, in which the award may be non-binding. We examine sealed-bid auctions with Pay-as-bid Pricing, Uniform Pricing with highest accepted bid (HAB) and lowest rejected bid (LRB) as well as the Vickrey rule. In the non-binding mechanism, bidders have the option to submit multiple bids and reject their award after the auction allocation. In this case, a reallocation procedure awards the best losing bid

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<sup>2</sup>See German Ground-mounted Photovoltaic Project Tender Act (Freiflächenausschreibungsverordnung), February 6th, 2015 (BGBl. I p. 108), 12 (3).  
Online (Law): <http://www.gesetze-im-internet.de/bundesrecht/ffav/gesamt.pdf>  
Online (results of 1st German PV Auction): <http://www.bundesnetzagentur.de>

(possibly another bid of the withdrawing bidder herself) and the award prices are adjusted correspondingly.

For the non-binding mechanism, we show that with symmetric and risk-neutral bidders, there exist pure strategy Bayesian Nash equilibria in monotone bidding functions under all pricing rules. In these, for the Uniform Price Auction (LRB) and the Vickrey Auction, the non-binding awards have no impact on bidding behavior, i.e. the incentive compatibility persists and bidders submit no additional bids. But, in the equilibrium of the Pay-as-bid Auction and of the Uniform Price Auction (HAB), if costs of rejection are neglected, bidders submit as many bids as feasible and the non-binding bids spread around the corresponding binding bid. However, we show that binding and non-binding mechanisms are both efficient and expected revenue equivalent. Our analysis is thereby based on Myerson (1981) and Engelbrecht-Wiggans (1988) who prove that feasible auction mechanisms that have the same allocation functions generate the same expected auction revenue.

We conduct an experiment to test the theoretically predicted bidding behavior and auction results. In many features, our experimental design strongly resembles the German PV Support Auction. The non-binding mechanism is compared to the well-known mechanism with binding awards, where bidders submit only one binding bid and no reallocation procedure takes place. Thus, we examine the non-binding mechanism under Pay-as-bid Pricing and Uniform Pricing with highest accepted bid, which are both implemented in the German PV Support Auction and compare it to the well-established binding mechanism.

Our experiment was conducted with z-Tree (Fischbacher, 2007) in February and March 2015. In total, 96 (paid) subjects participated in 8 sessions and 4 treatments — binding Pay-as-bid, non-binding Pay-as-bid, binding Uniform and non-binding Uniform. Each subject participated only under one pricing rule, i.e. either Pay-as-bid Pricing or Uniform Pricing, but in both respective variations, i.e. binding awards and non-binding awards. In each of the 40 periods of our procurement auction, each subject independently draws her private production costs and competes for the sale of one unit of 2 homogenous goods against 2 other subjects in a stranger matching within cohorts of 6 subjects. In each auction 2 homogenous goods are auctioned, i.e. the bids of 2 of 3 competing subjects in each round are awarded. The binding treatments correspond to the well-known auction mechanisms, whereas in the non-binding treatments, subjects are allowed to submit up to 2 alternative bids for their supply

of the single good.

Our experimental results support the theoretical hypotheses with respect to bidding behavior as well as to auction outcomes. We observe revenue equivalence between the binding and non-binding mechanism as well as between Pay-as-bid and Uniform pricing, as predicted by theory (see Figure 1). In the non-binding mechanism, subjects make use of the multiple

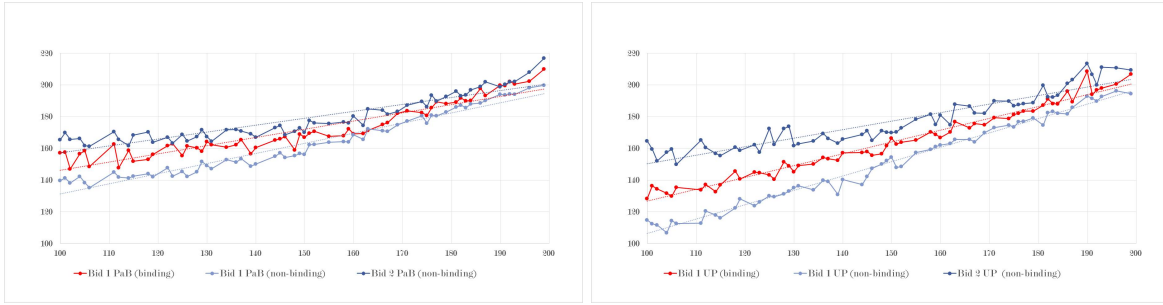
Pay-as-bid Auction			Uniform Price Auction		
Cohort	Binding	Non-binding	Cohort	Binding	Non-binding
C01	161.28	163.73	C09	165.80	164.35
C02	171.55	162.68	C10	164.45	163.45
C03	162.65	165.43	C11	166.35	168.45
C04	163.65	164.95	C12	164.50	164.25
C05	168.93	161.55	C13	167.65	168.40
C06	163.35	164.18	C14	166.85	153.15
C07	161.08	163.18	C15	159.85	166.80
C08	165.00	167.18	C16	165.10	163.55
<b>Total</b>	<b>164.68</b>	<b>164.11</b>	<b>Total</b>	<b>165.07</b>	<b>164.05</b>

Figure 1: Revenue equivalence: Auction revenue aggregated by cohorts.

bids option in both auction formats. We observe—in line with our hypothesis—in the non-binding auctions that, on average, a bidder’s lower bid is lower and her higher bid is higher than her corresponding bid in the binding auctions (see Figure 2). The data suggests that the non-binding mechanism facilitates the calculation of optimal bid-shading by offering the option of diversifying the bids, whereas the binding mechanism allows only one shot. That is, bidders submit more aggressive as well as less aggressive bids in the non-binding mechanism compared to the one-shot bid in the binding mechanism.

With respect to efficiency, all our treatments generate high efficiency rates of at least 97% (see Figure 3). However, we find that in terms of the number of efficient auction allocations the non-binding mechanisms perform significantly better than their binding counterparts. The underlying intuition for this result is based on the diversification of bids in the non-binding mechanism: a bidder’s lowest bid is nearer to her true costs than her bid in the corresponding binding auction. This leads to a more accurate mapping (regarding the monotonicity) of individual costs onto bids and, thus, better efficiency performance.

To summarize, we find both theoretical and experimental evidence that non-binding awards per se are not crucial for an auction to fail. Auctions rather seem to be highly



(a) Binding and non-binding bids (Pay-as-Bid Pricing) (b) Binding and non-binding bids (Uniform Pricing)

Figure 2: Bid Diversification: Aggregated bids as functions of induced values.

Pay-as-bid Auction			Uniform Price Auction		
Cohort	Binding	Non-binding	Cohort	Binding	Non-binding
C01	99%	98%	C09	95%	98%
C02	93%	93%	C10	97%	100%
C03	97%	97%	C11	99%	99%
C04	97%	97%	C12	99%	97%
C05	96%	96%	C13	99%	99%
C06	99%	99%	C14	97%	98%
C07	98%	98%	C15	99%	99%
C08	98%	98%	C16	99%	99%
<b>Total</b>	<b>97%</b>	<b>99%</b>	<b>Total</b>	<b>98%</b>	<b>99%</b>

Figure 3: Efficiency: Efficiency rates aggregated by cohorts.

sensitive to specific design elements as observed in the less successful CMS auction, where the non-binding awards are combined with a median pricing rule, as well as to the conditions under which they are implemented. Particularly, in case of a reallocation procedure, non-binding awards seem to have no unfavorable impact on multi-unit auctions with single-unit supply, neither under Pay-as-bid nor Uniform Pricing.

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