Discrimination in Auctions for Renewable Energy Support: Three Theoretically Equivalent but Practically Different Concepts

Karl-Martin Ehrhart\textsuperscript{1,1}, Marie-Christin Haufe\textsuperscript{1,1}, Jan Kreiss\textsuperscript{1,1},

\textsuperscript{a}Takon GmbH, Germany
\textsuperscript{b}Karlsruhe Institute of Technology (KIT), Germany

Abstract

The design of auctions for renewable energy support becomes more complex by the integration of different types of bidder into the same auction. This particularly applies to auctions in which bidders with asymmetric cost structures participate, e.g., bidders with different technologies in technology-neutral auctions or bidders from different countries in cross-border auctions. In order to privilege specific bidder groups and to control the allocation, discriminatory elements are included into the auction design. We analyze the three most applied discriminatory instruments: a minimum quota or a bonus for a bidder class to be privileged or different maximum prices for different bidder classes. Typically, these instruments discriminate stronger bidders (with lower costs) in favor of weaker bidders (with higher costs). We show that all three instruments can reduce the support costs in comparison to free competition by applying the principle of monopolistic third-degree price discrimination. Moreover, we prove that the three instruments are theoretically equivalent: every auction outcome that can be reached by one instrument can also be reached by the others including the outcome with minimal support costs. However, there are crucial differences concerning the practical application, particularly

\textsuperscript{*}Corresponding author

\textit{Email addresses: ehrhart@kit.edu} (Karl-Martin Ehrhart), \textit{haufe@takon.com} (Marie-Christin Haufe), \textit{kreiss@takon.com} (Jan Kreiss)
with respect to the robustness to misestimations of the cost structures. We show that the combination of the instruments helps to avoid costly errors. Finally, we illustrate our analyses by an example.

1. Introduction

Competitive bidding processes are globally becoming the instrument of choice regarding the promotion of renewable energies (RE). Auctions have proven to reduce the costs of RE support, increase efficiency, and control the RE expansion in order to reach the respective targets (Wigand et al., 2016). Therefore, the European Commission requires its member states from 2017 on to conduct auctions for RE support (European Commission, 2014). Moreover, the European Commission also proposes to conduct auctions that are open to multiple RE technologies. This requirement is based on the assumption that multi-technology auctions increase efficiency and reduce support costs even further. Multi-technology auctions have been implemented, e.g., in the United Kingdom (Department for Business, Energy & Industrial Strategy, 2017), Spain (Ministerio de Energía, Turismo y Agenda Digital, 2017a), the Netherlands (Minister van Economische Zaken, 2015) or Mexico (Centro Nacional de Control de Energía, 2017). Denmark and Germany conducted auctions for large PV installations that were open to bidders from both countries (Kitzing and Wendring, 2016).

Although there is a vast consensus about the use of auctions for RE support, national governments pursue different targets with their auctions, particularly with respect to the type of costs that are aimed to be minimized (Kreiss et al., 2017). There are arguments to address only support costs or only generation costs or also to include integration costs (Joskow, 2011; Ueckerdt et al., 2013). Most commonly, the minimization of support costs

\footnote{Concerns about windfall profits are sometimes cited as arguments against auctions with heterogeneous types of bidders, e.g., multi-technology auctions (Held et al., 2006).}
is the stated primary goal and main reason for RE auctions, e.g., in the United Kingdom (Department of Energy and Climate Change, 2011), in Mexico (Centro Nacional de Control de Energía, 2017) and in California (Public Utilities Commission of the State of California, 2010) it is explicitly stated that the auctions should minimize the costs of RE support. The definition of auction goals in other countries and even the statement of the European Commission can be interpreted so that the support costs are (one of) the most important target (European Commission, 2014; Ministerio de Energía, Turismo y Agenda Digital, 2017b). That is, the minimization of support costs attracts particular attentions when designing auctions for RE support.

Auctions for RE support also include discriminatory design elements to privilege specific bidder groups and to control the allocation. The focus of this paper is to analyze different discriminatory instruments and their effects on the auction outcome, particularly on support costs. The considered instruments are minimum or maximum quotas, maximum prices (i.e., reservation prices), and boni or mali for different bidder classes. All these instruments have been implemented in auctions for RE support: different maximum prices in the multi-technology auction in the Netherlands (Minister van Economische Zaken, 2015), a bonus depending on the location in the German auction for onshore wind (Deutscher Bundestag, 2016) and quotas that depend on the availability in the Californian auctions (Public Utilities Commission of the State of California, 2010).

The implementation of discriminatory instruments is often not only intensified by the minimization of the support costs but also by other criteria, e.g., grid and system integration, mixture of different RE technologies, regional distribution of RE, or actor diversity (Kreiss et al., 2017). Also in the context of such criteria, it is important to understand the effects of the discriminatory instruments on the auction outcome and the support costs.
The effects of discriminatory instruments implemented in markets and especially in auctions have been theoretically analyzed in a general context by, e.g., Schmalensee (1981), Varian (1989), Myerson (1981), Bulow and Roberts (1989), and McAfee and McMillan (1989). This paper goes one step further with a detailed analysis of the three discriminatory instruments for the actual application of RE auctions.

We show that each instrument (quota, bonus, maximum price) can reduce the support costs for RE sources, and we derive optimality conditions (w.r.t. support cost minimization) for each instrument and prove that the instruments are theoretically equivalent: every auction outcome (including support costs) that can be implemented by a specific parameterization of one instrument can also be reached by the two other (correspondingly parameterized) instruments.

However, with respect to the application in practice, there are crucial differences between the instruments, which have to be taken into account when deciding on their implementation and calibration. This particularly refers to the robustness of the desired effects of discrimination and the risk and magnitude of undesired effects that may be caused by a wrong calibration, e.g., due to misestimation of the absolute and relative strength of the different bidder classes that are treated differently in the auction. This is also of particular interest in the above mentioned cases with additional or other targets than the minimization of support costs.

This paper transfers microeconomic theory to a dynamic environment of increasing importance. It helps to understand the effects that different discriminatory instruments have on bidding behavior and the auction outcome. Since auctions for RE support become increasingly relevant and more and more auctions are opened for several technologies or participants from different countries, the relevance of this topic also increases.

In Section 2 we introduce the framework of our theoretic analysis of auctions for RE
support. The discriminatory instruments of a quota, maximum price, and a bonus are analyzed in Sections 2.1, 2.2, and 2.3. An illustrating example is provided in Section 2.4. The results of our analyses are compared and discussed regarding the practical implementation in Section 3. We summarize in Section 4.

2. Model

Consider a procurement auction for RE support with a fixed demand $D$ for a specific good (e.g., capacity [MW] or energy [MWh per year]). The supply side is modeled by single-project bidders each offering the same volume share that sum up to the total supply. The bidders have independent and private project costs for producing an unit of the good. The ascending order of private project costs is given by the marginal cost function $MC(x)$ with $MC(0) > 0$ and $MC'(x) = \frac{dMC(x)}{dx} > 0$ for all $x \geq 0$. That is, if $x$ is delivered by the projects with the lowest costs, $MC(x)$ are the highest marginal costs among these projects. The lowest total costs $C(x)$ for delivering $x$ are given by the cumulated marginal costs, $C(x) = \int_0^x MC(z)dz$. In the context of RE, $MC(x)$ are the levelized costs of electricity (LCOE) at $x$, i.e., the net present value of the total life cycle costs per unit of generated electricity of the RE source which would be ranked in the ascending order at $x$ (Short et al. 1995). Hence, $C(x)$ are the aggregated LCOE for delivering $x$ of all RE sources with LCOE lower than or equal to $MC(x)$.

There are two disjoint classes of bidders (e.g., two different technologies): low-cost bidders $(L)$ and high-cost bidders $(H)$. The two bidder classes $L$ and $H$ are characterized by different marginal cost functions $MC_L$ and $MC_H$ with

$$MC_L(x) < MC_H(x) \text{ for all } x \geq 0.$$  

That is, the marginal costs and total costs for delivering any volume $x$ are lower for
the low-cost bidders than for the high-cost bidders. In the context of RE, this means that the high-cost bidders need higher support for delivering a certain volume $x$ than the low-cost bidders. According to [IRENA (2015)], different RE sources in different countries and different years have significantly different cost structures. This means, costs to supply RE in a specific country and year are lower for one technology than for another. However, the overall costs might be minimized utilizing both technologies\textsuperscript{2}.

In the auction, the uniform price rule is applied and the uniform price is determined by the lowest rejected bid. Bidders simultaneously submit their bids for the monetary support for their projects. In this auction, a bidder’s optimal bidding strategy (weakly dominant strategy) is to bid the support that exactly covers his costs \textsuperscript{3}. Therefore, the supply functions $S_L(p)$ and $S_H(p)$ of the low-cost and high-cost bidders are given by

$$S_L(p) = MC_L^{-1}(p) \quad \text{and} \quad S_H(p) = MC_H^{-1}(p) \quad (2)$$

and increase in the price $p$. From (1) follows

$$S_L(p) > S_H(p) \quad \text{for all} \quad p \geq MC_L(0). \quad (3)$$

Thus, in free competition, the market clearing price $p^*$ is determined by

$$S_L(p^*) + S_H(p^*) = D, \quad (4)$$

where the supply of the low-cost bidders exceeds the supply of the high-cost bidders:

\textsuperscript{2}Even though the marginal costs for every demand $x$ are lower for one technology, there are demands $y$ and $\tilde{y}$ with $\tilde{y} > y$ so that the marginal costs for the lower cost technology for demand $\tilde{y}$ are higher than for the higher cost technology and the lower demand $y$, i.e., $MC_H(y) < MC_L(\tilde{y})$.

\textsuperscript{3}Since the marginal cost functions and, thus, the supply functions are continuous, there is no difference between the price rule of the lowest rejected bid and the price rule of the highest accepted bid, which is more common in practice.
\( S_L(p^*) > S_H(p^*) \geq 0 \). The auctioneer’s total costs amount to \( K(p^*) = p^*D \), i.e., the overall costs of all support payments to the awarded RE projects.

The elasticities of supply of the two bidder classes \( L \) and \( H \) are defined as

\[
\epsilon_i(p) = \frac{S'_i(p)}{S_i(p)} \quad \text{with} \quad S'_i(p) = \frac{dS_i(p)}{dp}, \quad i \in \{L, H\}. \tag{5}
\]

Additional to (1), we state the following assumptions.

Assumption 1.

(i) The elasticities of supply \( \epsilon_L(p) \) and \( \epsilon_H(p) \) are non-increasing in \( p \).

(ii) \( S_H(p^*) > 0 \) and \( \epsilon_L(p^*) < \epsilon_H(p^*) \) at the market clearing price \( p^* \) of free competition.

Assumption (i) is a standard economic assumption and also supported by the RE literature (de Vries et al. 2007; Hoefnagels et al. 2011; Brown et al. 2016). Assumption (ii) is more context sensitive: the high-cost bidders at least gain a small share in a non-discriminatory auction. Since this share is smaller than that of the low-cost bidders, it is reasonable to assume that the high-cost bidders’ price elasticity of supply at \( p^* \) is higher than that of the low-cost bidders.

In the following, we analyze and compare three commonly discussed and implemented instruments of discrimination. First, a quota is granted to the high-cost bidders in order to guarantee a certain minimum amount delivered by them. Second, a maximum (reservation) price for the low-cost bidders is set, which the low-cost bidders

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\(^4\)In case that the bidder classes are distinguished by their technology, there are examples of regions where one technology is much less costly than the other so that the high-cost bidders never have a chance to be awarded in a non-discriminatory auction. This, for example applies to North Dakota (Brown et al. 2016) or to Norway (Hoefnagels et al. 2011), where wind energy is much less costly than PV. However, there are many examples where wind and solar are both awarded in multi-technology auctions, e.g. Mexico (IRENA 2017), or are awarded in separate auctions but at similar price levels, e.g. in Germany (Bundesnetzagentur 2017a,b).
must not exceed with their bids. Third, the high-cost bidders receive a bonus in form of an additional payment in case of award. All three forms of discrimination induce a (supply) volume shift from the low-cost to the high-cost bidders (by always covering the total auction volume $D$) involving a respective price change. Moreover, all three forms of discrimination necessarily involve different prices $p_L$ and $p_H$ for the awarded low-cost and the awarded high-cost bidders.

In our analyses, we consider the total support costs

$$K(p_L, p_H) = p_L S_L(p_L) + p_H S_H(p_H)$$

(6)

which depend on the prices $p_L$ and $p_H$ and the corresponding supply volumes $S_L(p_L)$ and $S_H(p_H)$ with $S_L(p_L) + S_H(p_H) = D$.

2.1. Quota

Consider a minimum quota $Q < D$ (sometimes referred to as minimum contingent) for the high-cost bidders\(^5\). The quota guarantees that the high-cost bidder group will at least supply $Q$. Thus, the low-cost bidders’ supply never exceeds $D - Q$. The quota only becomes effective if $Q > S_H(p^*)$, i.e., the high-cost bidders would not reach $Q$ in free competition. If the quota becomes effective, each bidder class gets its own uniform award price $p_L$ and $p_H$, which are given by

$$p_L = MC_L(D - Q) \quad \text{and} \quad p_H = MC_H(Q).$$

(7)

The volume shift

$$q = \max\{Q - S_H(p^*), 0\}$$

(8)

\(^5\)Analogously, we could consider a maximum quota for the low-cost bidders.
from the low-cost bidders to the high-cost bidders, induced by the quota $Q$, increases the award price for the high-cost bidders and decreases the award price for the low-cost bidders compared to free competition, $p_H < p^* < p_L$, if $q > 0$. The volume shift $q$ and the price difference both effect the support costs. First, we investigate the effects by starting at $q = 0$, which corresponds to the situation of an ineffective quota.

**Lemma 1.** Consider a procurement auction with uniform pricing, fixed demand $D$, two bidder classes $L$ and $H$ and a minimum quota $Q$ for the high-cost bidders $H$. The support costs decrease when the quota becomes effective, i.e., $q = \max\{Q - S_H(p^*), 0\}$ becomes positive.

Lemma 1, whose proof is presented in Appendix A, states that the auctioneer can reduce costs by limiting the low-cost bidders. This is due to the properties of the elasticities of supply. If the high-cost bidders’ elasticity of supply exceeds that of the low-cost bidders at the free competition price $p^*$, then the relative price change (savings) induced by a marginal (negative) volume change for the low-cost bidder group is greater than the relative price change (costs increase) induced by a marginal (positive) volume shift of the high-cost bidders. In other words, higher prices for the (few) high-cost bidders increase the overall cost less than the overall cost reduction through lower prices for the (many) low-cost bidders. Consequently, the auctioneer can reduce the support costs by implementing a quota that leads to $q > 0$. Based on this result, we prove the existence of an optimal quota and its uniqueness.

**Proposition 1.** There exists an unique quota $\hat{Q} > S_H(p^*)$ that minimizes the support costs. The optimal quota $\hat{Q}$ together with the award prices $p_L$ and $p_H$ are determined by $\hat{Q} = S_H(p_H)$, $S_L(p_L) + S_H(p_H) = D$ and

$$p_H - p_L = \frac{S_L(p_L)}{S'_L(p_L)} - \frac{S_H(p_H)}{S'_H(p_H)},$$

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The proof is presented in Appendix B. The price difference \( p_H - p_L \) between high-cost and low-cost bidders that is induced by the optimal quota is also referred to in the literature. McAfee and McMillan (1989) show that this applies to international auction where domestic and foreign companies compete. Both results are based on the principle of monopolistic third degree price discrimination (Schmalensee, 1981; Varian, 1989). The monopolist discriminates different classes to absorb the different spending power. In the context of auctions for RE support, the auctioneer absorbs profits from the low-cost bidders and reduces the bidder rent at the expense of an inefficient outcome. The total support decreases as long as the elasticity of supply of the high-cost bidders is larger than that of the low-cost bidders.

That is, the auctioneer not only has an incentive to implement a quota under mentioned conditions to reduce his costs but there also exists an optimal quota that minimizes the expected support costs.

2.2. Maximum Price

The next discriminatory instrument is a reservation price \( r \) in form of a maximum price for the low-cost bidders: low-cost bidders may not submit bids higher than \( r \). The maximum price does not have an effect on bidding behavior and, thus, incentive compatibility holds: the bidders of both classes bid their true costs, except for the low-cost bidders with higher individual costs than \( r \), who do not participate.

The maximum price becomes effective if \( r < p^* \), i.e., the maximum price is lower than the uniform award price in free competition. Then, by (2), the low-cost-bidders receive a smaller volume \( S_L(r) < S_L(p^*) \) and a lower price \( r < p^* \) than in free competition and, by (4), the high-cost bidders receive a higher volume and, by (2), a higher price. That is, the maximum price induces a volume shift from the low-cost bidders to the high-cost bidders and a higher price \( p_H \) for the high-cost bidders than the price \( p_L = r \) for the low-cost bidders. These effects are equivalent to the effects of a volume shift (8).
induced by a quota \( Q > S_H(p^*) \), as derived in Section \( 2.1 \).

**Corollary 1.** Consider a procurement auction with uniform pricing, fixed demand \( D \), two bidder classes \( L \) and \( H \) and a maximum price \( r \) for the low-cost bidders \( L \).

(i) The support costs decrease when the maximum price \( r \) becomes effective, i.e., \( r - p^* \) becomes negative.

(ii) There exists an unique maximum price \( \hat{r} > 0 \) that minimizes the support costs. The optimal maximum price \( \hat{r} \) together with the award price \( p_H \) for the high-cost bidders is determined by \( S_L(\hat{r}) + S_H(p_H) = D \) and

\[
p_H - \hat{r} = \frac{S_L(r)}{S'_L(\hat{r})} - \frac{S_H(p_H)}{S'_H(p_H)}.
\]

2.3. Bonus

First, we consider a bonus in form of an additional monetary payment to the awarded high-cost bidders.\(^6\) Let \( b > 0 \) denote the bonus that is added to the award price \( p \) for the awarded high-cost bidders.

Incentive compatibility holds in the sense that the low-cost bidders bid their true costs and the high-cost bidders reduce their bid by exactly the bonus \( b \) (Thiel, 1988). Thus, all bidders receive their costs if the award price equals their bid. However, the awarded low-cost bidders receive \( p_L = p \), while the awarded high-cost bidders receive \( p_H = p + b = p_L + b \).

The higher price \( p_H \) for the high-cost bidders leads to a corresponding supply increase for these bidders described by (2). Together with the analysis in Section \( 2.1 \), this directly implies the following. First, implementing a bonus \( b > 0 \) for the high-cost

\(^6\)Analogously, we could consider a malus for the low-cost bidders in form of a deduction on the award price.
bidders is equivalent to a volume shift from the low-cost bidders to the high-cost bidders that is induced by a quota $Q > S_H(p^*)$. Second, the bonus that fulfills the condition in Proposition minimizes the support costs. We can state the following result.

**Corollary 2.** Consider a procurement auction with uniform pricing, fixed demand $D$, two bidder classes $L$ and $H$ and a bonus $b$ for the high-cost bidders $H$.

(i) The support costs decrease when the bonus $b$ becomes positive.

(ii) There exists an unique bonus $\hat{b} > 0$ that minimizes the support costs. The optimal bonus $\hat{b}$ together with the award price $p$ is determined by $S_L(p) + S_H(p + b) = D$ and

$$\hat{b} = \frac{S_L(p)}{S'_L(p)} - \frac{S_H(p + b)}{S'_H(p + b)}.$$  

The second bonus type is the so-called bid bonus, which reduces the high-cost bidders’ bids by $b$. This reduction is only relevant in competition and does not apply to the award price of the high-cost bidders. The bid bonus “strengthens” the bids of the high-cost bidders and, thus, increase their chance of being awarded. Incentive compatibility holds for both bidder classes, i.e., all bidders bid their true costs. Due to the bid bonus, the supply of the high-cost bidders $S_H(p_H)$ and their award price $p_H$ are higher than in the free competition case, while the reverse holds for the low-cost bidders. Since the argumentation is the same as for the monetary bonus, Corollary also applies to the bid bonus.

**2.4. Example**

The following example illustrates the principle of functionality of the three discriminatory instruments $\hat{Q}$, bonus $\hat{b}$ and maximum price $\hat{r}$. In our example, we assume that the marginal costs of the bidders in class $i \in \{L, H\}$ are uniformly distributed
over the interval \([a_i, b_i]\) with \(a_H > a_L > 0\), \(b_H > b_L\), and \(b_L > a_H\). We first assume 
\(b_H - a_H = b_L - a_L\) and the same number of bidders in the two classes. This leads to 
linear marginal cost functions of the form
\[
MC_i(x) = \frac{x}{m} + a_i
\]
(9)
for \(x \in [0, m(b_i - a_i)]\) with \(m > 0\). The number of bidders is represented by \(m\), i.e., the 
inverse of the gradient of the marginal cost function, which, by assumption, is the same 
in the two classes. Thus, the marginal cost functions are parallel shifts of each other. 
Translated to a practical application this means that there are as many low-cost bidders 
as high-cost bidders, however, there is a structural price difference \((a_H - a_L)\) between 
the two groups. Thus, the condition of lower costs for every quantity is fulfilled. By 
(2), the supply functions for \(i \in \{L, H\}\) are
\[
S_i(p) = \begin{cases} 
0 & \text{for } p < a_i \\
m(p - a_i) & \text{for } p \geq a_i.
\end{cases}
\]
(10)
with
\[
S'_i(p) = \frac{dS_i(p)}{dp} = m.
\]
(11)
for \(p > a_H\), which holds by Assumption I (ii). By (5), the elasticity of supply is
\[
\varepsilon_i(p) = \frac{S'_i(p)}{S_i(p)} p = \frac{p}{p - a_i},
\]
(12)
which is non-increasing for all \(p \geq 0\),
\[
\frac{d\varepsilon_i(p)}{dp} = - \frac{a_i}{(p - a_i)^2} < 0 \text{ for all } p \neq a_i.
\]
Since $a_H > a_L$, $\varepsilon_H(p) > \varepsilon_L(p)$ for all $p > a_H$. In free competition, the uniform award price $p^*$ is determined by the market clearing condition (4) and (10):

$$p^* = \frac{D}{2m} + \frac{a_L + a_H}{2}. $$

This leads to

$$S_H(p^*) = m(p^* - a_H) = \frac{D}{2} - \frac{m}{2}(a_H - a_L),$$

$$S_L(p^*) = m(p^* - a_L) = \frac{D}{2} + \frac{m}{2}(a_H - a_L).$$

That is, both bidder classes receive one half of the total demand plus/minus a spread that depends on the difference $(a_H - a_L)$ and the gradient $m$. Part (ii) of Assumption 1 requires

$$\frac{D}{m} > a_H - a_L. \quad (13)$$

The total support costs are

$$K(p^*) = K_L(p^*) + K_H(p^*) = p^* S_L(p^*) + p^* S_H(p^*) = p^* m (2p^* - a_L - a_H) \quad (14)$$

$$= Dp^* = \frac{D}{2} \left( \frac{D}{m} + a_L + a_H \right). \quad (15)$$

In Figure 1 the individual support costs $K_L(p^*)$ and $K_H(p^*)$ are visualized by the areas $p^* S_L(p^*)$ and $p^* S_H(p^*)$. Clearly, the low-cost bidders receive a larger payment as they supply more.

We now consider the optimal quota $\hat{Q}$, bonus $\hat{b}$, and maximum price $\hat{r}$. According to Proposition 1 and Corollary 1 and 2, optimal values of discriminatory instruments
are determined by the price difference

\[ p_H - p_L = \frac{S_L(p_L)}{S'_L(p_L)} - \frac{S_H(p_H)}{S'_H(p_H)} = (p_L - a_L) - (p_H - a_H), \]

which directly determines the optimal bonus \( \hat{b} \):

\[ p_H - p_L = \frac{a_H - a_L}{2} = \hat{b}. \]  

(16)

Applying (14), the relationship between the free competition price \( p^* \) and the prices \( p_H \) and \( p_L \) under the optimal discriminatory instruments is given by

\[ S_L(p^*) + S_H(p^*) = S_L(p_L) + S_H(p_H) = D, \]

\[ \Rightarrow 2p^* = p_H + p_L. \]
With (16), we get

\[
p_H = p^* + \frac{a_H - a_L}{4} = \frac{D}{2m} + \frac{3a_H + a_L}{4},
\]

\[
p_L = p^* - \frac{a_H - a_L}{4} = \frac{D}{2m} + \frac{3a_L + a_H}{4} = \hat{r},
\]

which also determines the optimal maximum price \(\hat{r}\).

The price increase for the high-cost bidders is equal to the price reduction for the low-cost bidders.\(^7\) By (10), (17) and (18), the supply volumes and the optimal quota \(\hat{Q}\) are

\[
S_H(p_H) = m \left( p^* - \frac{3a_H + a_L}{4} \right) = \frac{D}{2} - \frac{m}{4} (a_H - a_L) = \hat{Q},
\]

\[
S_L(p_L) = m \left( p^* - \frac{3a_L + a_H}{4} \right) = \frac{D}{2} + \frac{m}{4} (a_H - a_L) = D - \hat{Q}.
\]

With (19) and (20), the volume shift \(q = \hat{Q} - S_H(p^*)\) from the low-cost bidders to the high-cost bidders is

\[
q = m \left( p^* - \frac{3a_H + a_L}{4} \right) - m(p^* - a_H) = \frac{m}{4} (a_H - a_L).
\]

Comparing the total support costs \(K(p_L, p_H)\) under the optimal quota \(\hat{Q}\), bonus \(\hat{b}\) and maximum price \(\hat{r}\) with \(K(p^*)\) in free competition (14) yields

\[
K(p_L, p_H) = p_H S_H(p_H) + p_L S_L(p_L)
\]

\[
= \left( p^* + \frac{a_H - a_L}{4} \right) m \left( p^* - \frac{3a_H + a_L}{4} \right) + \left( p^* - \frac{a_H - a_L}{4} \right) m \left( p^* - \frac{3a_L + a_H}{4} \right)
\]

\[
= K(p^*) - \frac{m}{8} (a_H - a_L)^2 < K(p^*).
\]

\(^7\)This equality is caused by the characteristics of the example because the marginal cost curves of both classes are parallel shifts of each other. This equality does not necessarily hold for other marginal cost curves.
The support costs $K(p_L, p_H)$ are lower than $K(p^*)$ by \( \frac{m}{8} (a_H - a_L)^2 \). In Figure 2, the individual support costs are visualized by the areas $p_H S_H(p_H)$ and $p_L S_L(p_L)$. Compared to Figure 1, the sum of the two areas, i.e., the total support costs, is smaller. Although the price increase for the high-cost bidders is equal to the price reduction for the low-cost bidders, the overall costs for the auctioneer decrease as the number of bidders for which the price increases is lower than the number of bidders for which the price decreases.

![Figure 2: Illustration of the example with optimal discriminatory instruments \( \hat{Q}, \hat{b} \) and \( \hat{r} \).](image)

3. Assessment and Practical Application of the Instruments

We proved that the implementation of a quota, a bonus, or a maximum price to discriminate the low-cost bidders in favor of the high-cost bidders reduces the overall support costs compared to a non-discriminatory auction. The three instruments are considered theoretically equivalent because all outcomes (including the same support cost minimum) that can be achieved by one instrument can also be achieved by the
others. Moreover, the three instruments have in common that a “too aggressive” level of discrimination may reverse the effect and increase the support costs even above the level of a non-discriminatory auction. However, there are differences concerning the practical implementations. In the following, we compare the three instruments with respect to their robustness to misestimations. In Section 3.3, we extend the example of Section 2.4 by including uncertainties regarding the marginal cost functions.

3.1. Robustness to Misestimations

As the exact number and strength of the bidders and thus their cost functions are usually unknown to the auctioneer, he has to calibrate the discriminatory instruments according to his beliefs and estimations. In the following, we analyze and compare the effects of misestimations, particularly on support costs, under the three different instruments.

Generally, a too low minimum quota for the high-cost bidders or a too high maximum price for the low-cost bidders may not have any effect, while a bonus is always effective.

For a more detailed analysis, we first consider the case that the auctioneer overestimates the costs of the high-cost bidders, i.e., the high-cost bidders are stronger than expected. Then, a quota does not have a negative effect because it is calibrated to high-cost bidders which are assumed to be weaker than they actually are. The same holds for the maximum price. In both cases, the calibration of the instrument is not optimal, but the costs are weakly lower than in free competition. A bonus, however, might over-privilege the high-cost bidders and, thus, increases the costs compared to free competition. This effect becomes stronger if the anticipated high-cost bidders are even stronger than the anticipated low-cost bidders. In this case, a quota or maximum price are ineffective, whereas a bonus discriminates in favor of the stronger bidders and, thus, increase the support costs.
Second, if the high-cost bidders’ costs are underestimated, the negative effect of a bonus is lower than the negative effect of a quota or of a maximum price. The bonus might not be sufficient for enough high-cost bidders to be awarded. The quota and maximum price, however, will lead to an inappropriate share of awarded high-cost bidders.

As a matter of course, the arguments in the last two paragraphs analogously hold if the costs of the low-cost bidders are overestimated or underestimated. We also discuss these two cases in the example in Section 3.3. In this section, we also analyze the effects of a wrong estimation of the size of a bidder class, i.e., the number of bidders within a class. In this case, a bonus is more robust to misestimations than a quota or a maximum price, as long as the general cost difference between the two classes is estimated correctly. Under quota and maximum price, the negative effect of misestimations is stronger if the number of high-cost bidders is overestimated than the other way round.

3.2. Combination of Discriminatory Instruments

The combination of discriminatory instruments can increase the robustness to misestimations. A good example is the combination of a bid bonus and a maximum quota: the high-cost bidders are privileged by a bid bonus, which, however, is only applied to a limited number of high cost bidders – those with the lowest bids. This number is determined by the maximum quota. Hence, the quota restricts the number of privileged bidders. In this way, the quota protects the auctioneer from excessive costs in case he overestimated the number or he strength of high-cost bidders.

This application reveals an important different between the monetary bonus and the bid bonus. As pointed out in Section 2.3, the two bonus types are to be considered equivalent if the bidders know that they will be privileged by the bonus. However, this is not the case here because the high-cost bidders do not know this when submitting
their bid. Since a bid bonus has no impact on the optimal bid (i.e., bidding the true cost), whereas a monetary bonus induces bidders to reduce their bid by the bonus, the bid bonus is the right choice for this application.

The combination of a bid bonus and a quota increases the robustness to misestimations compared to the two instruments alone as it combines their advantages and lessens their disadvantages. The effectiveness of the discriminatory instrument is guaranteed, while possible negative side-effects are limited. This also applies to other combinations. Thus, the advantages of discriminatory instruments in auctions with different bidder classes can be utilized without further information regarding the bidders’ strength and number.

These considerations particularly apply to cases in which the implementation of discriminatory instruments is also guided by other targets than the minimization of support costs, such as grid and system integration, mixture of different RE technologies, regional distribution of RE, or actor diversity (Kreiss et al. 2017). Here, the considerations about the combination of instruments can help to keep costs low and to project against “unpleasant surprises.”

3.3. Illustrative Example of Consequences of Misestimations

In this section we illustrate the effects and implications of misestimations by means of the example of Section 2.4.

First, we examine the case in which the auctioneer does not correctly estimate the relation between the number of high-cost bidders and the number low-cost bidders. In line with our assumptions in Section 2.4, we model the relation between the number of bidders by introducing parameter \( \lambda \) into the marginal cost function (9) of the high-cost bidders,

\[
MC_H^\lambda(x) = \frac{\lambda}{m}x + a_H
\]

(22)
for \( x \in [0, \frac{m}{\lambda}(b_H - a_H)] \) with \( MC_H(x) \in [a_H, b_H] \) and \( \lambda > 0 \). Since we consider only two classes, it is sufficient to only change the marginal cost function of one class. Thus, the low-cost bidders’ marginal cost function\(^9\) remains \( MC_L(x) = \frac{x}{m} + a_L \) with \( MC_L(x) \in [a_L, b_L] \) for \( x \in [0, m(b_L - a_L)] \).

In the following we assume that the auctioneer estimates \( \lambda = 1 \) as in the example in Section 2.4. Thus, every actual \( \lambda \neq 1 \) refers to a situation in which the auctioneer’s estimate is wrong. For \( \lambda > 1 \) there are less and for \( \lambda < 1 \) there are more high-cost bidders than the auctioneer expects\(^8\). The actual supply function yields

\[
S_H^\lambda(p) = \begin{cases} 
0 & \text{for } p < a_H, \\
\frac{m}{\lambda}(p - a_H) & \text{for } p \geq a_H.
\end{cases}
\] (23)

Note, by (12), the elasticity of supply \( \varepsilon_i(p) = \frac{p}{p-a_i} \) does not depend on \( \lambda \), which is due to the linear supply function.

In the the free competition case, the equilibrium price and the supply volumes are

\[
p^\lambda = \frac{D}{m} \frac{\lambda}{\lambda + 1} + \frac{\lambda a_L + a_H}{\lambda + 1},
\]

\[
S_H^\lambda(p^\lambda) = \frac{D}{\lambda + 1} - \frac{m}{\lambda + 1}(a_H - a_L),
\]

\[
S_L^\lambda(p^\lambda) = \frac{D\lambda}{\lambda + 1} + \frac{m}{\lambda + 1}(a_H - a_L),
\] (24)

which yields the total support costs

\[
K^\lambda(p^\lambda) = Dp^\lambda = \frac{\lambda}{\lambda + 1}D \left( \frac{D}{m} + a_L + \frac{a_H}{\lambda} \right).
\] (25)

That is, the price and the support costs decrease in \( \lambda \).

\(^8\)The combination of the parameters \( \lambda, a_H, a_L, b_H, b_L \) is assumed to be such that the curves of the two classes do not intersect in the considered interval.
Second, we also consider the case in which the auctioneer does not correctly estimate the general relation between the strength of the two classes, given by the difference between \( a_H \) and \( a_L \).

In the following, we investigate the effects of misestimations of \( \lambda \) and \( a_H - a_L \) on the calibration of the discriminatory instruments and the support costs.

The optimal bonus, which by Corollary 2 is determined by the price difference \( p_H^\lambda - p_L^\lambda \), is given by

\[
\hat{b}^\lambda = p_H^\lambda - p_L^\lambda = \frac{S_L(p_L^\lambda)}{S_L'(p_L^\lambda)} - \frac{S_H(p_H^\lambda)}{S_H'(p_H^\lambda)} = \frac{a_H - a_L}{2}.
\]

(26)

Thus, the optimal bonus does not depend on \( \lambda \) and is equal to the optimal bonus in (16) for \( \lambda = 1 \). In the linear case, the bonus is robust to misestimations regarding the number of bidders and still leads to the support cost minimum. However, by (26), the optimal bonus \( \hat{b}^\lambda \) depends on \( a_H - a_L \) and, thus, is not robust with respect to misestimations of the general cost difference of the bidder classes.

Things are different for the optimal maximum price \( \hat{r}^\lambda \) for the low-cost bidders. By (17), (18), and (23), the optimal award prices \( p_L^\lambda \) and \( p_H^\lambda \) for the two classes and \( \hat{r}^\lambda \) are given by

\[
p_H^\lambda = \frac{D}{m \lambda + 1} + \frac{(2 + \lambda) a_H + \lambda a_L}{2(\lambda + 1)},
\]

\[
p_L^\lambda = \frac{D}{m \lambda + 1} + \frac{(2\lambda + 1) a_L + a_H}{2(\lambda + 1)} = \hat{r}^\lambda.
\]

That is, \( \hat{r}^\lambda \) depends on \( \lambda \) and, as shown in Appendix C, increases in \( \lambda \).

The effect of misestimations regarding the number of bidders is the same for the
optimal quota $\hat{Q}^\lambda$, which with (19), (20), and (23) is determined by

$$S_H(p_H^\lambda) = \frac{D}{\lambda+1} - \frac{m}{2(\lambda+1)}(a_H - a_L) = \hat{Q}^\lambda, \quad (27)$$

$$S_L(p_L^\lambda) = \frac{D\lambda}{\lambda+1} + \frac{m}{2(\lambda+1)}(a_H - a_L) = D - \hat{Q}^\lambda.$$ 

The optimal quota $\hat{Q}^\lambda$ depends on $\lambda$ and, as shown in Appendix C, decreases in $\lambda$.

Hence, the implementation of the maximum price $\hat{r}$ yields the same result as the implementation of $\hat{Q}$. That is, a misestimation regarding $\lambda$ has the same effect if either a quota or a maximum price is implemented.

Misestimations of the difference $a_H - a_L$ have the same effect for the optimal maximum price $r^\lambda$ and the optimal quota $Q^\lambda$. The volume shift induced by both instruments

$$q^\lambda = \frac{m}{2(\lambda+1)}(a_H - a_L),$$

which is given by the difference between (27) and (25) and which depends on $\lambda$ and $a_H - a_L$. Hence, maximum price and quota are also not robust regarding misestimations of the general relation between the strength of the bidder classes.

If the auctioneer implements the quota $\hat{Q}$, which in this case is equivalent to the implementation of the maximum price $\hat{r}$, the support costs depend on the misestimation of $\lambda$. To see this, consider $K^\lambda(Q)$, i.e., the support costs depending on a quota $Q$:

$$K^\lambda(\hat{Q}) = (D - \hat{Q}) \cdot MC_L(D - \hat{Q}) + \hat{Q} \cdot MC_H^\lambda(\hat{Q}) \geq K^\lambda(\hat{Q}^\lambda). \quad (28)$$

The equality only holds for $\lambda = 1$. If $\lambda \neq 1$, the costs $K^\lambda(\hat{Q})$ of the non-optimal quota $\hat{Q}$ are higher than the costs $K^\lambda(\hat{Q}^\lambda)$ of the optimal quota $\hat{Q}^\lambda$ and, as shown in Appendix C, the difference $K^\lambda(\hat{Q}) - K^\lambda(\hat{Q}^\lambda)$ increases in $|\lambda - 1|$. 

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For \( \lambda < 1 \), \( K^\lambda(\hat{Q}^\lambda) \leq K^\lambda(\hat{Q}) \leq K^\lambda(0) \). That is, discrimination through \( \hat{Q} \) or \( \hat{r} \) still leads to lower costs than in the free competition case with \( Q = 0 \) although the auctioneer wrongly estimates the relation of the bidder classes (see Appendix C). This does not hold for \( \lambda > 1 \). In this case, \( K^\lambda(\hat{Q}) > K^\lambda(0) \) is possible, i.e., the implementation of a non-optimal quota or maximum price leads to higher costs than in the free competition case without discrimination.

As shown before, the bonus \( \hat{b} \) is (more) robust to wrong estimation of \( \lambda \). In the linear model, the optimal bonus does not depend on \( \lambda \) but only on \( a_H - a_L \). The key to design a discriminatory auction robust to misestimations of \( \lambda \) is to achieve the optimal cost difference (26) which is always fulfilled through the optimal bonus.

Figure 3 illustrates the effect of either the implementation of \( \hat{Q} \) or \( \hat{r} \) on the basis of different misestimations of \( \lambda \). The optimal price difference \( p_H - \hat{r} \) is only implemented if the relation of the number of high-cost bidders and low-cost bidders is estimated correctly. For \( \underline{\lambda} < 1 \) the price difference is lower than \( p_H - \hat{r} \) and, thus, there is...
potential for further cost reductions. For $\bar{\lambda} > 1$ it is possible that the costs are even higher than in the free competition case.

4. Conclusion

It is a general trend in the expanded implementation of auctions for RE support to open the auctions to either multiple technologies or to bidders from several countries. The corresponding buzzwords are “technology-neutral” and “cross-border” auctions. As a consequence, designing the auction becomes more complex and more stakeholders express their opinion and try to shift the design parameters in their favor. Obviously, the argumentation that more competition always reduces prices falls short. The most discussed topics regarding more open auction formats for RE support are dynamic efficiency, integration costs and windfall profits.

We contribute to this discussion by transferring general microeconomic principles to the RE auction applications. We showed how three different discriminatory auction design elements – a quota, a bonus and a maximum price – can be implemented in auctions for RE support and what their implications are. We proved for each instrument that the discrimination of the stronger bidders in favor of the weaker bidders reduces the overall support costs. Moreover, to formulate it the other way round, if the introduction of a discriminatory instrument does not reduce the support costs, then there is no sense in conducting a non-discriminatory, multi-technology auction because only the strong technology would be awarded.

Additionally, we proved that the support cost minimum can be achieved by each of the three instruments if implemented in the optimal way. However, the optimal implementation of a quota, a bonus, or a maximum price requires information regarding the cost distribution of the different bidder groups. Depending on the availability of this information the three instruments vary regarding their robustness.
That is, if the auctioneer aims to minimize the support costs, the auction design should include discriminatory elements. Note, however, that this cost reduction is at the expense of efficiency. While the effective discrimination reduces the overall support costs, the awarded bidders are not necessarily those with the lowest generation costs. This conflict between support cost minimization and efficiency highlights the importance for the auctioneer to be aware of his targets and their priority.

There is no panacea for designing the “right” auction for the promotion of RE sources. The design has to be adapted to the target and the current market and technological developments, possibly including discriminatory instruments.

Appendix A. Proof of Lemma 1

Proof. Let $\Delta(q)$ denote the change in the support costs induced by $q$ compared to free competition, which by (6) and (7) is

$$
\Delta(q) = MC_L(S_L(p^*) - q) \cdot (S_L(p^*) - q) + MC_H(S_H(p^*) + q) \cdot (S_H(p^*) + q) - K(p^*).
$$

Differentiating $\Delta(q)$ with respect to $q$, denoted by $\Delta'(q)$, yields

$$
\Delta'(q) = -MC'_L(S_L(p^*) - q)(S_L(p^*) - q) - MC'_L(S_L(p^*) - q)
+ MC'_H(S_H(p^*) + q)(S_H(p^*) + q) + MC_H(S_H(p^*) + q).
$$

Starting with an ineffective quota, $q = 0$, to prove that the costs decrease when the quota becomes effective, we have to show that

$$
\Delta'(0) = -MC_L(S_L(p^*)) - S_L(p^*) MC'_L(S_L(p^*)) + MC_H(S_H(p^*)) + S_H(p^*) MC'_H(S_H(p^*)) < 0,
$$
i.e., the support cost change is negative and thus the costs decrease.

By $MC_L(S_L(p^*)) = MC_H(S_H(p^*)) = p^*$, we obtain

$$\Delta'(0) = S_H(p^*)MC'_H(S_H(p^*)) - S_L(p^*)MC'_L(S_L(p^*)) < 0.$$ \hspace{1cm} (A.1)

With $MC'_i(S_i(p)) = \frac{1}{S'_i(p)}$ for $i \in \{L, H\}$, (A.1) becomes

$$\frac{S_L(p^*)}{S'_L(p^*)} > \frac{S_H(p^*)}{S'_H(p^*)} \iff \frac{S'_L(p^*)}{S_L(p^*)}p^* < \frac{S'_H(p^*)}{S_H(p^*)}p^*$$

By (5), this condition is fulfilled if

$$\varepsilon_L(p^*) < \varepsilon_H(p^*),$$

which is given by Assumption 1 (ii).

Appendix B. Proof of Proposition 1

Proof. Consider the support costs

$$K(p_L, p_H) = p_L S_L(p_L) + p_H S_H(p_H) \text{ with } S_L(p_L) + S_H(p_H) = D.$$ \hspace{1cm} (B.1)

The minimization of the Lagrange function of (B.1) with regard to $p_L$ and $p_H$ yields the first order conditions

$$\frac{\partial K(p_L, p_H)}{\partial p_L} = S_L(p_L) + p_L S'_L(p_L) + \lambda S'_L(p_L) = 0$$

$$\frac{\partial K(p_L, p_H)}{\partial p_H} = S_H(p_H) + p_H S'_H(p_H) + \lambda S'_H(p_H) = 0$$
which lead to the condition

\[ p_H - p_L = \frac{S_L(p_L)}{S'_L(p_L)} - \frac{S_H(p_H)}{S'_H(p_H)}. \]  \hspace{1cm} (B.2)

For \( Q \leq S_H(p^*) \), \( p_H = p_L = p^* \) and, thus, the left-hand side of (B.2) is zero. \( Q > S_H(p^*) \) implies \( p_H > p^* > p_L \). With an increasing \( Q \), \( p_H \) increases and \( p_L \) decreases and, thus, the left-hand side of (B.2) increases. (5) and Assumption 1 (ii) imply that the right-hand side of (B.2) is positive at \( p^* \) which demonstrates that the optimality condition (B.2) does not hold for an ineffective quota, e.g. \( Q \leq S_H(p^*) \). By Assumption 1 (i), \( \varepsilon_H(p_H) \) does not increase and \( \varepsilon_L(p_L) \) does not decrease if \( p_H \) increases and \( p_L \) decreases. Thus, with (5), the right-hand side of (B.2) decreases. Since the left-hand side of (B.2) increases with an increasing quota \( Q \) and the right-hand side of (B.2) decreases, there exist a unique \( \hat{Q} \) that fulfills (B.2) and Assumption 1 (ii) that an increasing quota \( Q \) above \( S_H(p^*) \) reduces the support costs, \( \hat{Q} \) is the unique cost minimum.

\[ \square \hspace{1cm} \square \]

Appendix C. Additional Calculations for the Extended Example in Section 3.3

To show that the equilibrium price \( p^\lambda \) increases if there are less high-cost bidders than expected, i.e., \( \lambda > 1 \), we calculate

\[ p^\lambda - p^* = \frac{\lambda - 1}{(\lambda + 1)^2} \left( \frac{D}{m} + a_L + a_H \right) \]

where \( (\frac{D}{m} + a_L + a_H) > 0 \) due to (13) and \( \frac{\lambda - 1}{(\lambda + 1)^2} \) is greater than zero for \( \lambda > 1 \) and negative for \( \lambda < 1 \). Therefore, the equilibrium price increases for an increasing \( \lambda \).

As a direct result, also the support costs \( K^\lambda(p^\lambda) \) are greater than \( K(p^*) \) if there are less high-costs bidders and vice versa with more high-cost bidders.

To prove that the implementation of a quota \( \hat{Q} \) in a case where \( \lambda \neq 0 \) is not optimal,
we have to show that the difference $\hat{Q}^\lambda - \hat{Q} \neq 0$:

$$\hat{Q}^\lambda - \hat{Q} = \frac{1 - \lambda m}{\lambda + 1} \left( \frac{D}{m} - \frac{1}{2}(a_H - a_L) \right)$$

where again $(\frac{D}{m} - \frac{1}{2}(a_H - a_L)) > 0$ due to (13) and

$$\frac{1 - \lambda}{\lambda + 1} \begin{cases} 
> 0 & \text{if } \lambda < 1 \\
= 0 & \text{if } \lambda = 1 \\
< 0 & \text{if } \lambda > 1 
\end{cases}$$

so that only for $\lambda = 1$ both quotas are identically. Moreover, for $\lambda > 1$ the optimal quota is lower than before and for $\lambda < 1$ it is the other way round.

Finally we calculate for $\lambda \neq 0$ and the optimal discriminatory auction the support costs

$$K^\lambda(\hat{Q}^\lambda) = \frac{D^2(\lambda^2 + 1)}{m(\lambda + 1)^2} + \frac{D}{\lambda + 1}(\lambda a_L + a_H) - \frac{m}{4(\lambda + 1)}(a_H - a_L)^2$$

and compare them to the costs given $\lambda \neq 0$ and a discriminatory auction with quota $\hat{Q}$

$$K^\lambda(\hat{Q}) = \frac{D^2(\lambda + 1)}{4m} + \frac{D}{4}((3 - \lambda)a_H + (1 + \lambda)a_L) - \frac{m}{16}(a_H - a_L)^2(3 - \lambda)$$

which results in the difference

$$K^\lambda(\hat{Q}^\lambda) - K^\lambda(\hat{Q}) = \frac{D^2(\lambda - 1)(\lambda^2 + 3)}{4m(\lambda + 1)^2} - \frac{D}{4(\lambda + 1)}(a_H - a_L)(\lambda - 1)^2 + \frac{m}{16(\lambda + 1)}(a_H - a_L)^2(\lambda - 1)^2$$

which is negative for all $\lambda > 0$ and only equals zero for $\lambda = 1$. The difference $K^\lambda(\hat{Q}) - K^\lambda(\hat{Q}^\lambda)$ increases in $|\lambda - 1|$. 

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