# Allocation, Prices, and Costs in the Electricity Wholesale Market and Balancing Power Market – An Integrated Approach

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**Abstract**

This paper considers the economic interdependencies of the electricity wholesale market and the balancing power market. We present an integrated market model that relates to the future harmonization of the European balancing power markets. We prove that there exists an integrated market equilibrium and analyze its properties. The comparison of our theoretical findings with empirical market data from Germany of 2015 reveals that the empirical costs decreased over the years but are still above the theoretical equilibrium. This holds particularly for the costs of balancing power activation, which is an indication that the suppliers achieved to establish a high price-level in the regular repeated balancing power auction.

*Keywords:* Balancing Power; Market Design; Market Equilibrium; Procurement Auction; Reserve Power
*JEL:* D2, D4, D5, D6, L1, L5

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## Notation

| | |
|---|---|
| $\alpha$ | gradient of function $S$ |
| $\beta$ | intercept of function $S$ |
| $\gamma^{+/-}$ | fraction of provided positive (negative) BP |
| $\delta$ | share of BP suppliers |
| $\vartheta$ | length of the BEPP |
| $\pi$ | supplier's profit function |
| $a^{+/-}$ | calling probability in the positive (negative) BP market |
| $a^{+/-}_{max}$ | highest calling probability in the positive (negative) market |
| $a^{+/-}_{min}$ | lowest calling probability in the positive (negative) market |
| $B$ | capacity demand on the positive and negative BP market |
| $B^{+/-}$ | capacity demand on the positive (negative) BP market |
| $\tilde{B}^{+/-}$ | expected positive (negative) capacity for balancing excess supply and demand |
| $C$ | total cost function of the power system |
| $[\underline{c}, \overline{c}]$ | interval of the variable energy production costs |
| $c^{+/-}_0$ | lowest variable cost of all suppliers on the positive (negative) BP market |
| $c^{+/-}_1$ | highest variable cost of all suppliers on the positive (negative) BP market |
| $c + dc$ | imputed variable cost |
| $CS$ | consumer surplus |
| $D$ | average demand on the wholesale market |
| $j$ | function of BP bidders' average active capacities |
| $J$ | BP bidders' average active capacities (integral of $j$) |
| $m$ | minimal load capacity |
| $P$ | normalized BE price |
| $p_S$ | wholesale market price |
| $p^{+/-}_{BP}$ | BP price in the positive (negative) market |
| $p^{+/-}_{BE}$ | BE price in the positive (negative) market |
| $p_{res}$ | reservation price for a reliable power system |
| $PS$ | producer surplus |
| $R^{+/-}$ | set of positive (negative) BP merit-order ranks |
| $r^{+/-}$ | rank function in the positive (negative) BP merit-order |
| $X$ | normalized position in the merit-order |
| $S$ | supply function |
| $S_{BP}$ | supply function of BP suppliers |
| $S_{nBP}$ | supply function of nBP suppliers |
| $Z$ | distribution function of the difference between demand and supply |

## 1. Introduction

The on-going penetration of renewable energy sources in the power market reduces the predicability of electric energy production substantially. To ensure a stable grid frequency (e.g., 50 Hertz in the synchronous grid of Continental Europe), ancillary services become increasingly important. The most relevant short-term ancillary service is balancing power (*BP*). In liberalized electricity markets, the procurement of BP is usually carried out by the Transmission System Operators (*TSOs*) or the regulatory authority. The applied market mechanism is mostly a procurement auction, in which prequalified suppliers compete for the provision of BP.

In the academic debate, there exist three approaches for the examination of BP markets. The first are empirical analyses. Rammerstorfer and Wagner (2009) provide an empirical assessment of the effects associated with a reorganization of the German BP market. Heim and Götz (2013) present evidence for strategic capacity withholding by a supplier with market power. Hirth and Ziegenhagen (2015) connect renewable energy sources to BP markets and discuss several implications, e.g., the increased volatility of electricity supply caused by solar and wind power plants. Ocker et al. (2016) and Ocker (2017) show that there is no prevalent market design in Europe, but a huge heterogeneity. Ocker and Ehrhart (2017) present empirical evidence for collusive behavior in the German BP markets.

The second are theoretical investigations of the BP auctions. Bushnell and Oren (1995) and Chao and Wilson (2002) analyze different scoring auctions and describe essential rules to ensure an efficient allocation. Kamat and Oren (2002) analyze efficiency properties of alternative auction formats for BP markets in the USA. Müller and Rammerstorfer (2008) categorize several auction elements that can be used for the design of BP markets.

The third approach belongs to BP market design and suppliers' behavior. Wen and David (2002) present a stochastic optimization model to derive optimal bidding strategies. Swider and Weber (2007) present a methodology for profit maximizing bidding under price uncertainty for power systems reserve. Müsgens et al. (2014) discuss the economic fundamentals that govern market design and behavior in the German BP markets. Ocker et al. (2018b) present bidding strategies in the Austrian and German BP auctions. Ortner (2017) discusses fundamental modelling approaches and illustrates case studies assuming perfect competition. Ocker et al. (2018a) present a game-theoretical analysis of the BP auctions and point out that the regular repetition of the auctions invite the suppliers to implicitly collude.

In most theoretical analyses, the interdependencies of the electricity wholesale market and the BP market are not appropriately considered. For example, Müsgens et al. (2014), Hirth and Ziegenhagen (2015) and Ocker et al. (2018a) assume that the wholesale market price is exogenous. This yields two classes of suppliers: suppliers with power plants that have variable cost below the wholesale market price, and suppliers with variable cost above the wholesale market price. The former sell electric energy profitably on the wholesale market, while the latter do not participate in this market. Müsgens et al. (2014) denote these two types of suppliers "inframarginal" and "extramarginal." This distinction has a direct impact on the costs for providing BP: inframarginal suppliers must integrate opportunity costs of not trading at the wholesale market, and extramarginal suppliers must cover their expenses by profits of the BP market (e.g. Hirth and Ziegenhagen, 2015; Ocker et al., 2018b).

To our knowledge, only Just and Weber (2008) address these interdependencies: suppliers of BP cannot offer their entire capacity on the wholesale market, however, have to run their plants at a certain minimal load. The authors focus on the identification of reservation pricing and the influence of reserve capacity on the supply function of the wholesale market. For this, they apply a numerical solution procedure.

Our approach relates to the work of Just and Weber (2008), however, we use a different methodology. We develop an integrated market model to analyze the interdependencies between the wholesale electricity market and the BP market. The interplay between the markets induces a specific assignment of the energy producers to the different markets according to the producers' production costs and their ability to provide BP. Thus, inframarginality and extramarginality are endogenously determined. There exists a unique market equilibrium that ensures efficiency under certain assumptions. We also consider prices and costs in the markets as well as the distribution of surpluses. The comparison with German market data reveals that the actual BP costs are higher than predicted by our model, which particularly applies to the costs of BP activation. Although the total costs gradually decreased over the last years, the gap between the predicted and observed costs of BP activation increased. We consider this as an indication that the suppliers successfully coordinated on a price-level above the theoretical equilibrium, which is facilitated by the regular repetition of the BP auction and the limited number of suppliers (Ocker et al., 2018b; Kaut et al., 2017).

The remainder of this paper is structured as follows. Section 2 provides a brief overview of the electricity wholesale and BP market design. Section 3 presents our integrated market model. In Section 4 we analyze and discuss the overall market equilibrium and the effects of the BP prequalification conditions and market power. Section 5 contrasts the theoretical findings with German market data. Section 6 concludes and points at further need of research.

## 2. Electricity Markets

In this section we provide a general overview of the electricity markets.

### *2.1. Electricity Wholesale Market*

Electric energy (henceforth energy) is traded at forward markets and at spot markets in most European countries (e.g. Ströbele et al., 2013; KU Leuven Energy Institute, 2015;

Zweifel et al., 2017). Forward markets enable the trade of energy more long-term, whereas at spot markets the point of delivery is instantaneous, i.e., typically within the next 48 hours. Therefore, forward markets are mostly utilized for risk hedging, and trading is often carried out bilaterally (so-called "over the counter"). When trading at spot markets, there are two types of markets available: a "Day-Ahead" auction and an "Intraday Continuous" auction.[1] In the Day-Ahead auction, trading is done for the following day, whereas in the Intraday Continuous auction, trading is done only minutes before the actual delivery. The market clearing price is commonly uniform and determined by the last (i.e., highest) accepted bid.

*2.2. Balancing Power Market*

The volumes traded at the wholesale market can differ substantially from the actual production of energy because they are based on predictions of supply and demand. A deviation directly influences the grid frequency in alternating current systems: if too much (little) energy is supplied, the grid frequency increases (decreases), which can lead to area-wide black-outs. For securing a reliable power system, most TSOs apply ancillary services for stabilizing the power grid. The most-important short-term ancillary service is BP (Müsgens et al., 2014; Hirth and Ziegenhagen, 2015; Zweifel et al., 2017).

There are three different qualities of BP: Primary BP (*PBP*), Secondary BP (*SBP*), and Tertiary BP (*TBP*) (e.g. entso-e, 2016; Ocker et al., 2016).[2] These are distinguished by the reaction time of BP being available: First, PBP is activated to limit deviations from the grid frequency, then SBP is utilized to restore the grid frequency and, finally, as along-term measure TBP is activated. The reaction times differ across Europe (e.g. Ocker et al., 2016). In Germany, for example, the reaction times are as follows: PBP must be available after 30 seconds until 5 minutes, SBP after 5 minutes until 15 minutes, and TBP after 15 minutes until 60 minutes after an imbalance. The three qualities have separate markets that are organized as procurement auctions (Ocker et al., 2016). We focus on the SBP market because it has the highest demand and is the most important short-term ancillary service (Borne et al., 2018). Since the SBP market is to be harmonized across Europe no later than 2021, we refer to the future European auction design in our analysis (European Commission, 2017).

A deviation of the predicted production schedule can either lead to an overproduction (e.g., from wind plants during a storm) or an underproduction (e.g., from solar plants during a cloudy day). Consequently, BP needs to provide both an increased and decreased energy supply. This is achieved by implementing two different BP products: in the positive market, suppliers provide upward regulation, while in the negative market, suppliers provide downward regulation (e.g. entso-e, 2016; Ocker et al., 2016).

[1] For an overview of the design of European spot markets see Ocker et al. (2016).

[2] PBP is also known as Frequency Containment Reserve (FCR), SBP as automatically-activated Frequency Restoration Reserve (aFRR), and TBP as manually-activated Frequency Restoration Reserve (mFRR).

Since suppliers of BP need to adjust the load level of their power plants within seconds, the provision of BP requires a high degree of operational and technical flexibility. Hence, an elaborate prequalification process for the BP market participation is required. As a consequence, the supply set is highly invariant and limited.[3]

Providing BP comes with costs for the suppliers: they have to be compensated for keeping BP (Megawatt, MW) available to the grid and also for the actual delivery of balancing energy ($BE$) (Megawatt hour, MWh). Suppliers submit three-dimensional bids in the SBP auction: a power offer (with unit MW), a power bid (with unit Euro/MW), and an energy bid (with unit Euro/MWh). Since TSOs are legally forced to procure BP at the lowest possible costs, they calculate scores for the suppliers and those with the lowest scores are awarded. In the future harmonized SBP market, TSOs may base these scores on both the power and energy bid, only the power bid or only the energy bid (European Commission, 2017).[4]

The awarded suppliers' power and energy price are determined via pricing rules. Two pricing rules are applied: Pay-as-bid ($PaB$) or uniform pricing ($UP$). If PaB is used, awarded suppliers are paid their submitted bids. If UP is used, all awarded suppliers are paid a uniform price. In the future harmonized SBP market, UP is to be implemented (entso-e, 2016).

For the determination of the uniform price (market clearing price), two rules are usually applied in practice: the price is determined either by the last (highest) accepted bid or by the first (lowest) rejected bid (e.g. Kahn et al., 2001; Müsgens et al., 2014; Ocker et al., 2018a). The fact that the first rule is more prevalent in practice should be interpreted as a convention. If a bidder's probability to determine the price with one of her bids is small, e.g. because the number of bidders is large, the strategic incentives are almost equal under both rules. Moreover, in an efficient auction equilibrium, both rules (and other pricing rules) lead to the same expected outcome, which includes the same allocation, the same expected bidders' profits, and the same expected auction revenue/costs (Engelbrecht-Wiggans, 1988).

The delivery of BE is activated according to a merit-order of the energy bids, which discriminates with respect to the activation duration of the power plants (European Commission, 2017). That is, the suppliers with the lowest energy bids are utilized first.

## 3. Integrated Market Model

In this section, we first illustrate how the markets interrelate and then present our model.

[3] For the German markets, see for example regelleistung.net (2017) for the prequalification criteria and Kaut et al. (2017) for an analysis of market concentration.

[4] In the theoretical analysis we relate to the scoring rule that considers only the power bid, since it is commonly applied across Europe (Ocker et al., 2016).

*3.1. Interdependencies*

BP is provided by prequalified suppliers that have to meet specific technical requirements, i.e., a certain degree of technical flexibility regarding the operation mode of their power plant.[5] Since not all types of power plants are qualifiable, there is a coexistence of two distinct types of suppliers on the wholesale market: suppliers that exclusively offer their capacities in the wholesale market, and suppliers which can offer their capacities on both the wholesale market and the BP market. Thus, there are fundamental interconnections between the two markets because BP suppliers cannot offer their entire capacity on the wholesale market, however, must run their power plants at a certain minimal load. This results in must-run capacities, whose energy is sold on the wholesale market (Just and Weber, 2008).

*3.2. The Model*

There are three energy markets: a wholesale market, a positive and a negative BP market. We consider a certain period (e.g. year). The average demand on the wholesale market in this period is denoted by $D$ and measured in Gigawatt (GW). The (capacity) demand on the positive and negative BP market is fixed and given by $B^+$ and $B^-$ (with $B = B^+ + B^-$).

There is a set of suppliers offering energy. Each supplier participates in the markets with one power plant. The suppliers' plants have the same capacity, but differ with respect to the variable energy production costs, which lie in the interval $[\underline{c}, \overline{c}]$. We assume that UP is implemented on all considered markets, which is the common pricing rule on the wholesale market and considers the intentions for a harmonized European SBP market (European Commission, 2017). That is, all awarded suppliers receive the marginal price. On the BP markets, the uniform price for BE is repeatedly determined within a pre-defined period of time – the "Balancing Energy Pricing Period" (*BEPP*). The implementation of UP allows us to assume that the suppliers reveal their cost in their bids.[6] Thus, the supply function $S : [\underline{c}, \overline{c}] \to \mathbb{R}^+$ is strictly increasing and $S(c)$ is the supply at price $c$. The inverse function is $S^{-1} : \mathbb{R}^+ \to [\underline{c}, \overline{c}]$.

[5]Depends on the criteria determined by the country, BP quality, etc. For Germany, see the official website of the four German TSOs (regelleistung.net, 2017).

[6]For the wholesale market, this is the usual and acceptable assumption. Things are different on the BP markets because here the time a supplier delivers BE depends on her merit-order position. As a consequence, the length of the BEPP has an impact on the suppliers' bidding strategy. The longer the BEPP, the more bids are taken into account for the determination of the price, which induces the suppliers to reduce their energy bids, even below their costs (Ocker et al., 2018a,b). Under the realistic condition that the last accepted bid determines the price under UP, a very short BEPP causes a high probability for a bidder that her awarded bid determines the price. In this case, which converges towards the PaB case, the suppliers have an incentive to exaggerate their costs in their bids. Thus, there exists a BEPP which incentivizes the suppliers to bid their costs. Nevertheless, applying the "Revenue Equivalence Theorem" (e.g. Krishna, 2002), the expected equilibrium outcome (i.e., allocation, suppliers' expected profits, expected (average) prices, and expected BP costs) neither depends on the pricing rule (PaB or UP) nor on the length of the BEPP under UP (see Ocker et al. (2018a) and Section 2.2). For these reasons and the sake of clarity and simplicity, we take the liberty to assume truthful bidding, i.e., the suppliers bid their true costs.

There are two types of suppliers: BP-capable ($BP$) suppliers and non-BP-capable ($nBP$) suppliers. The nBP suppliers only participate on the wholesale market, while BP suppliers can participate on the wholesale market and the BP market. For the latter, they must run their plant on a minimal load (i.e., share of capacity) $m \in [0, 1)$ and sell this energy on the wholesale market. The supply includes BP and nBP suppliers: $S(c) = S_{BP}(c) + S_{nBP}(c)$, where $S_{BP} : [\underline{c}, \overline{c}] \to \mathbb{R}^+$ and $S_{nBP} : [\underline{c}, \overline{c}] \to \mathbb{R}^+$ denote the strictly increasing supply functions of BP and nBP suppliers. We assume that BP suppliers are uniformly distributed among all suppliers: at each cost level $c$, the BP suppliers' share of the supply $S(c)$ is $\delta \in [0, 1]$ and, thus, the nBP suppliers' share is $1 - \delta$.

Discrepancies between demand and supply are balanced by calling BP. The function

$$Z : [-B^-, B^+] \to [0, 1] \tag{1}$$

describes the distribution of the difference between demand and supply in the period, where $Z(x)$ for $x < 0$ refers to excess supply and, thus, to the call of negative BP, while $Z(x)$ for $x \geq 0$ refers to excess demand and to the call of positive BP (see Figure 1). That is, $Z(x)$ for $x < 0$ ($x \geq 0$) is the probability of an excess supply (excess demand) of at least $|x|$ GW. We assume that the discrepancies are only caused by supply fluctuations due to production deviations.[7]

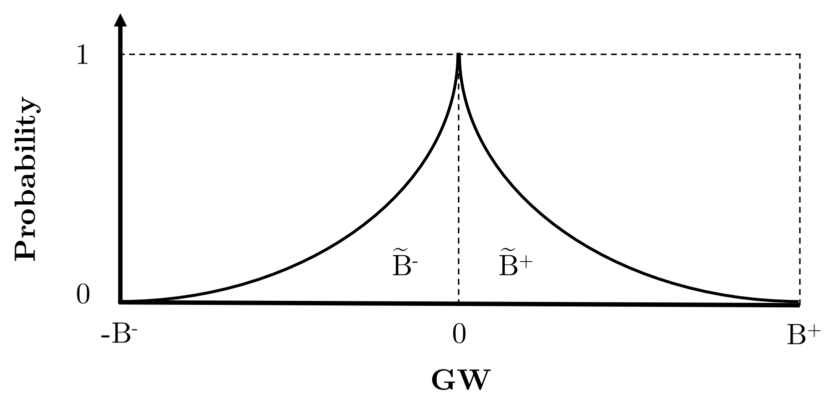


Figure 1: Visualization of the $z$-function.

Hence, (1) also describes the probabilities for calling BP (see Figure 4 in Section 5.2.2 as an example), i.e., $Z(x)$ for $x < 0$ ($x \geq 0$) describes the share of time within the period where a minimum capacity of $|x|$ negative (positive) BP is called. Function $Z(x)$ is strictly increasing for $x < 0$ and strictly decreasing for $x \geq 0$ with $Z(-B^-) = Z(B^+) = 0$ and $Z(x) + Z(x') \leq 1$

[7]This can be justified by the increasing penetration of variable renewable energy sources into the power system. Their energy production depends on the weather conditions and is therefore highly volatile.

for $x \in [-B^-, 0)$ and $x' \in [0, B^+]$. The integrals

$$\tilde{B}^- = \int_{-B^-}^{0} Z(x)dx \quad \text{and} \quad \tilde{B}^+ = \int_{0}^{B^+} Z(x)dx \tag{2}$$

are the expected total negative BP capacity and the expected total positive BP capacity that are needed to balance the excess supply and excess demand. Thus, $\gamma^- = \tilde{B}^-/B^-$ and $\gamma^+ = \tilde{B}^+/B^+$ are the fraction of provided negative BP and of positive BP, i.e., the fraction of the BP capacities for the delivery of BE demand (with $\tilde{B} = \tilde{B}^+ + \tilde{B}^-$).

We call the BP markets symmetric if the difference between demand and supply on the wholesale market is symmetrically distributed, i.e., $B^- = B^+ = \frac{B}{2}$ and $Z(-x) = Z(x)$ for $x \in (0, \frac{B}{2}]$. In this case, $\tilde{B}^- = \tilde{B}^+$ and the average demand and average supply are equal.

Let $c_0^+$ ($c_0^-$) denote the lowest variable cost of all suppliers on the positive (negative) BP market and $c_1^+$ ($c_1^-$) the highest variable cost.[8] The BP merit-order maps a supplier (according to her cost $c$) onto a merit-order position on the positive market by the bijective function $r^+ : [c_0^+, c_1^+] \to [0, B^+] =: R^+$ and on the negative market by $r^- : [c_0^-, c_1^-] \to [B^-, 0] =: R^-$. Each rank is assigned a calling probability by the mappings $a^+ : R^+ \to [a^+_{max}, a^+_{min}]$ and $a^- : R^- \to [a^-_{min}, a^-_{max}]$. The calling probability determines the average share of time in which the supplier delivers BE. The values $a^+_{max}$ and $a^-_{max}$ ($a^+_{min}$ and $a^-_{min}$) denote the highest (lowest) calling probability in the two BP markets. As mentioned above, the calling probabilities are determined by the $Z$-function (1), where

$$\begin{aligned} a^+_{min} &= Z(B^+) = 0, \\ a^+_{max} &= Z(0) \in (0, 1), \\ a^-_{min} &= Z(-B^-) = 0, \\ a^-_{max} &= \lim_{x \to 0^-} Z(x) \in (0, 1), \\ a^+_{max} &+ a^-_{max} \leq 1. \end{aligned} \tag{3}$$

On the wholesale market, suppliers constantly produce energy and are remunerated for each unit by the wholesale market price $p_S$. On the BP markets, a supplier receives the BP price $p^+_{BP}$ or $p^-_{BP}$, and, if she is called, additionally a BE price $p^+_{BE}(c)$ or $p^-_{BE}(c)$. The BP prices $p^+_{BP}$ and $p^-_{BP}$ are the same for all suppliers and are determined by the highest accepted power bid in the positive respectively negative BP market. The BE price $p^+_{BE}(c)$ ($p^-_{BE}(c)$)

[8]Any supply fluctuation demands the provision of BP. The respective costs are accounted to the suppliers. In our theoretical model, we do not account for these additional costs since they reflect on average approximately 0.1% of the suppliers variable cost (see Section 5.3). We assume that any supply fluctuation triggers SBP, and that all suppliers of the wholesale market deviate identically. For this, $dc$ denotes the average cost of BE per MW caused by supply fluctuations. The sum $c + dc$ represents the imputed variable cost.

is determined by the associated costs of the highest merit-order position that is needed to cover the demand within the BEPP. Thus, the length of the BEPP influences the supplier's BE prices: the longer the BEPP, the higher is the number of draws for BE demand, and, thus, the higher (lower) are the cost of the last supplier on the positive (negative) market. We model the average supplier's BE price in dependence of factor $\vartheta \in (0,1]$ that corresponds to the length of the BEPP:

$$p_{BE}^{+}(c) = c\,(1-\vartheta) + c_1^{+}\,\vartheta\,, \tag{4}$$

$$p_{BE}^{-}(c) = -c\,(1-\vartheta) - c_0^{-}\,\vartheta\,. \tag{5}$$

The case $\vartheta = 1$ models the longest possible BEPP, in which the BE price $p_{BE}^{+}$ ($p_{BE}^{-}$) is always determined by the highest (lowest) supplier's cost $c_1^{+}$ ($c_0^{-}$).[9] The smaller $\vartheta$ (i.e., the shorter the BEPP), the closer moves the supplier's average BE price to her cost $c$.

A supplier's profit per produced energy unit on the wholesale market is[10]

$$\pi_S(c) = p_S - c\,, \tag{6}$$

on the positive BP market

$$\begin{aligned}\pi_{BP}^{+}(c) &= m(p_S - c) + (1-m)p_{BP}^{+} + (1-m)a(r(c))(p_{BE}^{+}(c) - c)\\ &= m(p_S - c) + (1-m)[p_{BP}^{+} + a(r(c))(p_{BE}^{+}(c) - c)]\,,\end{aligned} \tag{7}$$

and on the negative BP market

$$\begin{aligned}\pi_{BP}^{-}(c) &= m(p_S - c) + (1-m)p_{BP}^{-} + (1-m)[(p_S - c) + a(r(c))(c + p_{BE}^{-}(c))]\\ &= (p_S - c) + (1-m)[p_{BP}^{-} + a(r(c))(c + p_{BE}^{-}(c))]\,.\end{aligned} \tag{8}$$

Equation (6) states the difference of the wholesale market price and a supplier's variable cost. In the positive BP market, the profits consist of two parts. The first part of (7) represents the profits from selling the minimal load on the wholesale market, and the second part states the profits generated by the BP price and the BE price (depending on the calling probability). In the negative BP market, suppliers are continuously paid the wholesale market price for their entire capacity. Recall that a supplier provides negative BP by decreasing the load level of her power plant, since there is an oversupply to the power system. Consequently, the provision of negative BP has no impact on her trading on the wholesale market. Therefore, the first part of (8) represents the margin of selling their entire capacity at the wholesale market. The

[9] Note that suppliers submit negative energy bids in the negative market (see (8)).

[10] The wholesale market price fluctuates and, thus, the supplier's profit, i.e., $p_S$ and $\pi_S(c)$ are average values.

second part states the BP profits, which consist of the payment for BP and BE.

### *3.3. Conditions for Efficiency, Stability and Market Clearing*

In this section we present some crucial conditions for our model.

#### *3.3.1. Efficient Allocation on the BP Markets*

An efficient allocation on the BP markets requires that plants with low variable cost are preferred to plants with high variable cost for the production of an additional unit of energy. Thus, plants with low variable cost must have higher production volumes than plants with high variable cost (Müsgens et al., 2014). This yields a unique order of $c_0^+$, $c_0^-$, $c_1^+$ and $c_1^-$.

In the negative BP market, $c_0^-$ denotes the supplier on the last rank in the merit-order with a calling probability of $a_{min}^- = 0$, i.e., she continuously produces with her entire capacity of size 1. The supplier with $c_1^-$ is assigned the calling probability of $a_{max}^-$, i.e., her plant operates with the load $m + (1 - a_{max}^-)(1 - m) < 1$. This yields $c_0^- < c_1^-$.

In the positive BP market, $c_0^+$ is assigned to the supplier on the first rank in the merit-order with calling probability $a_{max}^+$. Thus, her plant operates on the load level $m + a_{max}^+(1 - m) \leq m + (1 - a_{max}^-)(1 - m)$ because of $a_{max}^+ + a_{max}^- \leq 1$. This yields $c_1^- = c_0^+$. The supplier with $c_1^+$ is on the last rank of the merit-order. She never provides BP, but runs her plant permanently on the load level $m$ on the wholesale market. As a result, we get the following order.

$$\textbf{(A0) Rank of costs} \quad c_0^- < c_1^- = c_0^+ < c_1^+$$

Note that this order in conjunction with (4) and (5) yield $|p_{BE}^-| < p_{BE}^+$.

#### *3.3.2. Stability Criteria*

BP suppliers either participate at the wholesale market or the BP market. This raises the question about the stability of the efficient BP allocation. That is, do prices exist such that the suppliers are incentivized to choose the "right" position of their own accord?

The conditions for the suppliers on the first and last position in the merit-orders are most crucial. The supplier with $c_0^-$ has to be indifferent between her last position in the merit-order of the negative BP market and a switch to the wholesale market. The supplier with $c_1^+$ must be indifferent between her last position in the merit-order of the positive BP market and not participating at all. The suppliers with $c_1^-$ ($c_0^+$) need to be indifferent between a switch to the positive (negative) BP market. This leads to the following stability conditions.

**(M0) Between-market** $\quad (p_S - c_0^-) + (1-m)p_{BP}^- \stackrel{!}{=} p_S - c_0^-$

**(M1) Market-entrance** $\quad m(p_S - c_1^+) + (1-m)p_{BP}^+ \stackrel{!}{=} 0$

**(M2) BP-markets**
$$(p_S - c_1^-) + (1-m)(p_{BP}^- + a_{max}^-(c + p_{BE}^-(c)))$$
$$\stackrel{!}{=} m(p_S - c_0^+) + (1-m)(p_{BP}^+ + a_{max}^+(p_{BE}^+(c) - c_0^+))$$

If one of these conditions is violated, either producing suppliers have an incentive to switch markets or non-producing suppliers have an incentive to enter the BP market.

*3.3.3. Market Clearing and Energy Balance*

Since BP suppliers only use the share $1-m$ of their capacities to provide BP, their total capacity to cover $B$ has to be $\frac{B}{1-m}$. According to the considerations in 3.2 and 3.3.1, this capacity is provided by the BP suppliers with costs between $c_0^-$ and $c_1^+$. In order to cover $B^-$, a total capacity of $\frac{B^-}{1-m}$ is needed, which is provided by the BP suppliers with costs between $c_0^-$ and $c_1^-$. The BP suppliers in the subsequent cost interval $[c_0^+, c_1^+]$ with $c_0^+ = c_1^-$ together provide $\frac{B^+}{1-m}$ for covering $B^+$. Hence, the interval $[c_0^-, c_1^+]$ corresponds to cumulated BP capacities in the interval $[0, \frac{B}{1-m}]$, where $[c_0^-, c_1^-)$ corresponds to $[0, \frac{B^-}{1-m})$ and $[c_0^+, c_1^+]$ to $[\frac{B^-}{1-m}, \frac{B}{1-m}]$. Within the interval $[0, \frac{B^-}{1-m})$, the calling probability increases from $a_{min}^- = 0$ to $a_{max}^-$. Thus, since we are on the negative BP market, the suppliers' (expected) active capacities for providing energy for the BP markets and the wholesale market decrease from 1 to $1-(1-m)a_{max}^-$. Within the interval $[\frac{B^-}{1-m}, \frac{B}{1-m}]$, which applies to the positive BP market, the calling probability decreases from $a_{max}^+$ to $a_{min}^+ = 0$, and thus, the suppliers' active capacities decrease from $m+(1-m)a_{max}^+$ to $m$. Due to the condition $a_{max}^- + a_{max}^+ \leq 1$, $1-(1-m)a_{max}^- \geq m + (1-m)a_{max}^+$ and, thus, the curve of active capacities is strictly decreasing within $[0, \frac{B}{1-m}]$. Applying the $Z$-function (1) together with (3), the curve of the BP suppliers' active capacities for providing energy for the BP markets and the wholesale market is given by the strictly decreasing function $j(q)$:

$$j(q) = \begin{cases} m + (1-m)\,(1 - Z(q(1-m) - B^-)) & : q \in [0, \frac{B^-}{1-m}) \\ m + (1-m)\,Z(q(1-m) - B^-) & : q \in [\frac{B^-}{1-m}, \frac{B}{1-m}]\,. \end{cases} \tag{9}$$

Using the maximum calling probability $a_{max}^+$ leads to

$$j(q) = \begin{cases} 1 & : q = 0 \\ m + (1-m)\,a_{max}^+ & : q = \frac{B^-}{(1-m)} \\ m & : q = \frac{B}{1-m}\,. \end{cases} \tag{10}$$

The integral

$$J(m,B) = \int_0^{\frac{B}{1-m}} j(q)dq \tag{11}$$

is the average active capacity of all BP suppliers. With (2) we get

$$J(m,B) = \frac{B^-}{1-m} - \tilde{B}^- + \frac{mB^+}{1-m} + \tilde{B}^+ \,. \tag{12}$$

Figure 2 illustrates the symmetric case with $j(\frac{B}{2(1-m)}) = \frac{1+m}{2}$, $j(\frac{B}{1-m} - q) = 1 + m - j(q)$ for $q \in [0, \frac{B}{2(1-m)}]$, and

$$J(m,B) = \frac{B\,(1+m)}{2(1-m)} \,. \tag{13}$$

That is, in the symmetric case, $J(m,B)$ is independent of the shape of the $j$-curve.

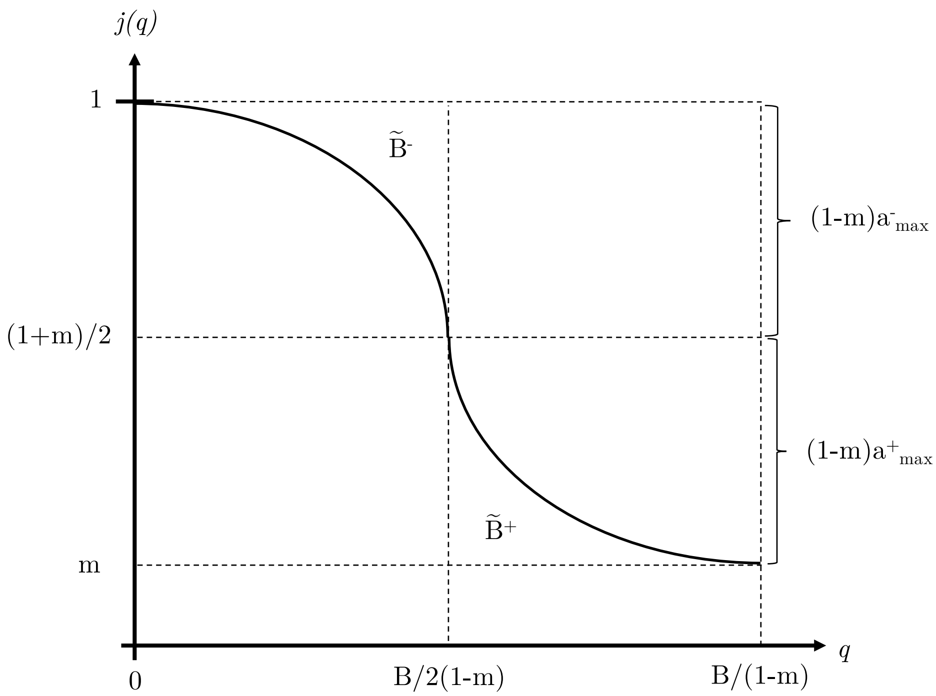


Figure 2: Example of a $j$-function in a symmetric BP market.

Market clearing and energy balance require the following conditions:

**(S0) Wholesale market** $D = S_{nBP}(p_S) + S_{BP}(c_1^-) + \frac{mB^+}{1-m}$

**(S1) Positive BP market** $B^+ = (1-m)(S_{BP}(c_1^+) - S_{BP}(c_0^+))$

**(S2) Negative BP market** $B^- = (1-m)(S_{BP}(c_1^-) - S_{BP}(c_0^-))$

**(S3) Energy balance** $D - S_{nBP}(p_S) + S_{BP}(c_0^-) + \tilde{B}^- - \tilde{B}^+ + J(m, B) = 0$

The demand $D$ on the wholesale market is met by the contracted supply, which refers to the case without deviations (S0). This supply is provided by nBP suppliers and BP suppliers. The supply $S_{nBP}(p_S)$ includes all nBP plants with cost between $\underline{c}$ and $p_S$. The supply of the BP plants comprises of two groups: the contracted supply of the negative BP suppliers with cost between $\underline{c}$ and $c_1^-$ is $S_{BP}(c_1^-)$ and that of the positive BP suppliers with cost between $c_0^+$ and $c_1^+$, whose contracts only refer to the minimal load $m$, is $\frac{mB^+}{1-m}$. The demand for positive and negative BP is provided by the $(1-m)$th share of BP plants within the interval $[c_0^+, c_1^+]$ and $[c_0^-, c_1^-]$ (S1, S2). Condition (S3) requires that total supply meets demand $D$ also in case of deviations. Positive deviations $\tilde{B^+}$ and negative deviations $\tilde{B^-}$ are balanced by the BP suppliers with cost between $c_0^-$ and $c_1^+$, whose active capacities are given by $J(m, B)$.

*3.4. Total Costs of the Integrated Power System*

The total costs $C$ of the integrated power system are

$$C = \int\limits_{(1-\delta)\beta}^{D-q_{BP}-q_0} S_{nBP}^{-1}(q)\, dq + \int\limits_{\delta\beta}^{q_0} S_{BP}^{-1}(q)\, dq + \int\limits_{0}^{\frac{B}{1-m}} j(q)\, S_{BP}^{-1}(q+q_0)\, dq\,, \tag{14}$$

with $q_0 = S_{BP}(c_0^-)$, $q_{BP} = J(m, B)$, $B = (1-m)(q_1 - q_0)$. Equation (14) includes the costs for the wholesale market and the BP markets. The first are determined by the costs of nBP suppliers in the interval $[0, D - q_{BP} - q_0]$ and the costs of BP suppliers in the interval $[0, q_0]$, while the costs for the BP markets are given by the costs of BP suppliers in the interval $[q_0, q_0 + \frac{B}{1-m}]$ weighted with the average active capacity of function $j(q)$.

*3.5. Welfare Distribution*

The producer surplus $PS$ is given by

$$PS = D\, p_S + B^+\, p_{BP}^+ + B^-\, p_{BP}^- - C\,.$$

$PS$ includes the profits of the wholesale market $D\, p_S$ and of the BP payments $B^+\, p_{BP}^+ + B^-\, p_{BP}^-$, while the total energy production costs $C$ are subtracted. The BE costs do not effect $PS$ because they are charged between the suppliers (see Section 3.2).

For the consumer surplus $CS$, we apply the concept of a consumers' reservation price (per energy unit) for a reliable power system $p_{res}$ (e.g. Zolotarev, 2017), which leads to

$$CS = D\,(p_{res} - p_S) - B^+\,p_{BP}^+ - B^-\,p_{BP}^-\,.$$

The consumer surplus $CS$ incorporates the difference of the reservation price $p_{res}$ and the wholesale market price $p_S$ for the demand $D$. The BP costs $B^+\,p_{BP}^+$ and $B^-\,p_{BP}^-$ reduce $CS$ because consumers bear the costs for keeping capacities available for BP.

Note that the variables $p_S$, $B^+$, $p_{BP}^+$, $B^-$, and $p_{BP}^-$ have opposed effects on $PS$ and $CS$: $PS$ increases and $CS$ decreases in each variable.

## 4. Market Equilibrium

The following propositions 1, 2, 3, and 4 are derived under the efficiency condition (A0), the micro-stability criteria (M1), (M2) and (M3), the conditions (S0), (S1), (S2) and (S3), symmetric BP markets, and a linear supply function

$$S(c) = \alpha\,c + \beta\,, \tag{15}$$

where $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$. Thus,

$$S_{BP}(c) = \delta(\alpha\,c + \beta)\,, \tag{16}$$

$$S_{nBP}(c) = (1-\delta)(\alpha\,c + \beta)\,. \tag{17}$$

**Proposition 1.** *There exists an equilibrium of the wholesale market and the BP markets with the following prices:*

1. *Wholesale market price:* $p_S = \dfrac{D-\beta}{\alpha} \leq m\,c_1^+ + (1-m)\,c_0^+$
2. *BP price in the positive BP market:* $p_{BP}^+ = \dfrac{m}{1-m}\,(c_1^+ - p_S) = \dfrac{\frac{B}{2}\,\frac{m}{1-m}}{\delta\,\alpha}$
3. *BP price in the negative BP market:* $p_{BP}^- = 0$

For the proof see AppendixA. The wholesale market price $p_S$ is determined by the inverse supply function at demand $D$. Condition $p_S \leq m\,c_1^+ + (1-m)\,c_0^+$ is necessary for stability since a higher $p_S$ induces suppliers of positive BP to switch into the wholesale market. The power price $p_{BP}^+$ in the positive BP market exactly covers the wholesale market loss of the supplier with $c_1^+ > p_S$ caused by her costs of supplying the minimal load $m$ and a calling probability of zero. The power price $p_{BP}^-$ in the negative BP market is zero (see also (M0)).

**Proposition 2.** *In the equilibrium of the wholesale market and both BP markets the following holds for $c_0^-$, $c_1^+$, $c_0^-$, and $c_1^+$:*

1. *The cost $c_0^-$ of the supplier on the last rank in the negative BP market is given by*

$$c_0^- = \frac{D-\beta}{\alpha} - \frac{\frac{B}{2}\frac{1+m}{1-m}}{\delta\,\alpha} = p_S - \frac{\frac{B}{2}\frac{1+m}{1-m}}{\delta\,\alpha} \leq p_S.$$

2. *The costs $c_1^-$ and $c_0^+$ of the suppliers on the first rank in both BP markets are given by*

$$c_1^- = c_0^+ = \frac{D-\beta}{\alpha} - \frac{\frac{B}{2}\frac{m}{1-m}}{\delta\,\alpha} = p_S - \frac{\frac{B}{2}\frac{m}{1-m}}{\delta\,\alpha} = p_S - p_{BP}^+ \leq p_S.$$

3. *The cost $c_1^+$ of the supplier on the last rank in the positive BP market is given by*

$$c_1^+ = \frac{D-\beta}{\alpha} + \frac{\frac{B}{2}}{\delta\,\alpha} = p_S + \frac{\frac{B}{2}}{\delta\,\alpha} \geq p_S.$$

For the proof see AppendixA. The cost $c_0^-$ of the first BP supplier is determined by the difference of the wholesale market price $p_S$ and the cost of the BP supply, which implies $p_S \geq c_0^-$. Costs $c_1^-$ and $c_0^+$ are equal and are determined by the difference of $p_S$ and the BP price in the positive BP market $p_{BP}^+$, which implies $p_S \geq c_1^- = c_0^+$. The cost $c_1^+$ of the last BP supplier is determined by $p_S$ and half of the costs of the entire BP supply, which implies $p_S \leq c_1^+$. Hence, all suppliers that provide negative BP are inframarginal, i.e., their cost are below the wholesale market price. This also applies to the lower part of the suppliers providing positive BP, while the suppliers of positive BP with higher cost are extramarginal, i.e., their cost are above the wholesale market price. According to Proposition 2 (2.), if the minimum capacity $m$ converges to zero, the line between suppliers of negative BP and those of positive BP, given by $c_1^-$, converges to the line between inframarginal and extramarginal bidders, given by the wholesale market price $p_S$. Thus, $c_1^- = c_0^+ = p_S$ for $m = 0$, i.e., all suppliers of negative BP are inframarginal and all suppliers of positive BP are extramarginal.

**Proposition 3.** *In the equilibrium of the wholesale market and both BP markets the following holds for the profits of the suppliers:*

1. *$\pi_S(c)$ and $\pi_{BP}(c)$ decrease in $c$.*
2. *$\pi_S(c) \geq 0$ for $c \in [\underline{c}, p_S]$ and $\pi_{BP}(c) \geq 0$ for $c \in [c_0^-, c_1^+]$.*
3. *$\pi_{BP}(c) \geq \pi_S(c)$, $\forall c \in [c_0^-, c_1^+]$.*

For the proof see AppendixA. Suppliers' profits decrease with their variable cost on all markets. The profits in all markets are (weakly) greater than zero, and a supplier's participation in the BP markets generates (weakly) higher profits than on the wholesale market.

**Proposition 4.** *The equilibrium of the wholesale market and both BP markets ensures overall market efficiency, i.e., it minimizes the total costs $C$ of the power system.*

For the proof see AppendixA. Here is an intuitive explanation. An upward shift of the interval $[c_0^-, c_1^+]$ has three effects: more expensive power plants provide BP, the supply of BP suppliers on the wholesale market increases, which crowds out nBP suppliers on this market. In the case of symmetric BP markets and a linear supply function, the three cost effects cancel each other out in the equilibrium. The reasons are that, due to the symmetric BP markets, the costs of BP supply only depend on $c_0^-$ and $c_1^+$ but not on $j(q)$, and, due to the linear supply function, the cost savings of negative BP (i.e., shutting down expensive power plants due to an overproduction of cheaper plants) equal the increasing costs for positive BP (i.e., activating more expensive plants due to an underproduction of cheaper plants).[11]

*4.1. Effect of BP Prequalification*

In this section we analyze the effects of less strict BP prequalification criteria, i.e., more flexibility for the suppliers to provide BP, on the equilibrium outcome, particularly on total costs (14). In our approach, suppliers' flexibility to offer BP is modelled by parameter $\delta$, i.e., the share of prequalified BP suppliers. An increasing $\delta$ (i.e., more flexibility) has the following effects: By Proposition 1, $p_{BP}^+$ decreases, while $p_S$ and $p_{BP}^-$ do not change. By Proposition 2, the interval $[c_0^-, c_1^+]$ becomes smaller because $c_0^-$ increases and $c_1^+$ decreases. The reduction of the interval length is caused by a higher density of BP suppliers. More precisely, the difference between $p_S = \frac{D-\beta}{\alpha}$ and $c_0^-$, between $p_S$ and $c_1^- = c_0^+$, and between $c_1^+$ and $p_S$ decreases. What is the effect of a higher $\delta$ on the total costs (14)?

**Proposition 5.** *The total costs* (14) *decrease when $\delta$ increases.*

The proof is presented in AppendixA and sketched in the following. In the equilibrium, the nBP suppliers and BP suppliers with the lowest costs run at full load. The BP plants with higher costs (i.e., $c > c_0^-$) run at a reduced load, and these loads continuously decrease – according to the $j$-function (9) – to the minimum load $m$ for the last active plant with the highest cost $c_1^+$. These nBP and BP suppliers together provide the required total capacity to cover $D$ and to provide BP. Since this capacity is independent of $\delta$, this also applies to the number of full load suppliers, which together provide $D$, and the number of suppliers with reduced load, which together provide BP. Hence, a higher density of BP suppliers and a smaller interval $[c_0^-, c_1^+]$ in combination with a higher $c_0^-$ and a lower $c_1^+$ and $c_0^- < p_S < c_1^+$ cause that full load nBP suppliers with high costs are replaced by full load BP suppliers with lower costs and that the BP suppliers with higher costs, who together provide BP with

[11] Note that in case of asymmetric BP markets or a non-linear supply function, efficiency cannot be guaranteed because the three cost effects do not necessarily cancel each other out in the equilibrium.

reduced load, move closer together with regard to their cost. Since the total provided capacity does not change with $\delta$, the total costs (14) decrease when $\delta$ increases.

With the focus on total costs, the results of this analysis recommend to increase flexibility by lowering the BP prequalification criteria.

### *4.2. Market Power*

The previous analysis is based on competitive markets, which is justified by the assumption of many suppliers, each with one power plant, which have the same capacity and only differ with respect to the variable energy production costs (see Section 3.2). While maintaining the assumption of the same capacity per plant, we model market power by a supplier that participates with multiple plants with the same or different variable costs.

Such a multiple-plant supplier has an incentive to submit different bids even for plants with the same costs (Ausubel et al., 2014; Krishna, 2002). While we can still assume that the supplier bids her true costs for her first plant with the lowest costs under UP, she has an incentive to exaggerate her costs for her further plants in order to increase the price and, thus, her profit.[12] The higher the supplier's market power (i.e., the more plants are in her portfolio), the stronger the incentive to exaggerate the costs in her further bids. As a consequence, the same number of plants distributed to fewer suppliers leads to higher prices. Since multiple-plant suppliers differentiate their bids, the market equilibrium may be inefficient.

Due to the required prequalification, fewer suppliers participate in the BP markets than in the wholesale market. Therefore, the incentive to exercise market power (i.e., stronger differentiation of the bids) is stronger in the BP markets. This leads to a further distortion of the outcome such that the price under market power in the BE markets exceeds the competitive price more than in the wholesale market. This difference may cause a transfer of market power from the wholesale market to the BP markets. Consider a BP supplier with multiple plants, whose costs are below $c_0^-$ and, thus, are only designated for the wholesale market in the competitive equilibrium. This supplier may has an incentive to switch to the BP markets by underbidding her competitive power bid in order to exercise market power on the BE market because this may lead to a higher expected profit than exercising market power in the more competitive wholesale market.

## 5. Comparison with German Market Data

In this section, we compare the data of the German SBP market of 2015 with the results of our theoretical model adapted to the German SBP market.[13]

[12] See Krishna (2002, Ch. 13) in combination with the argumentation in Section 2.2 and Footnote 6.

[13] We investigate the market results of 2015 for two reasons: First, 2015 is the most recent year for which publicly available data for SBP was made available by the German TSOs. Second, since July 2016, the

### 5.1. Asymmetric SBP markets

The German SBP markets are asymmetric, which is reflected by the shape of function $j(q)$ (see Figure 3). Now, $\gamma^+ \neq \gamma^-$ and $\tilde{B}^+ \neq \tilde{B}^-$. As a consequence, the effects of the positive

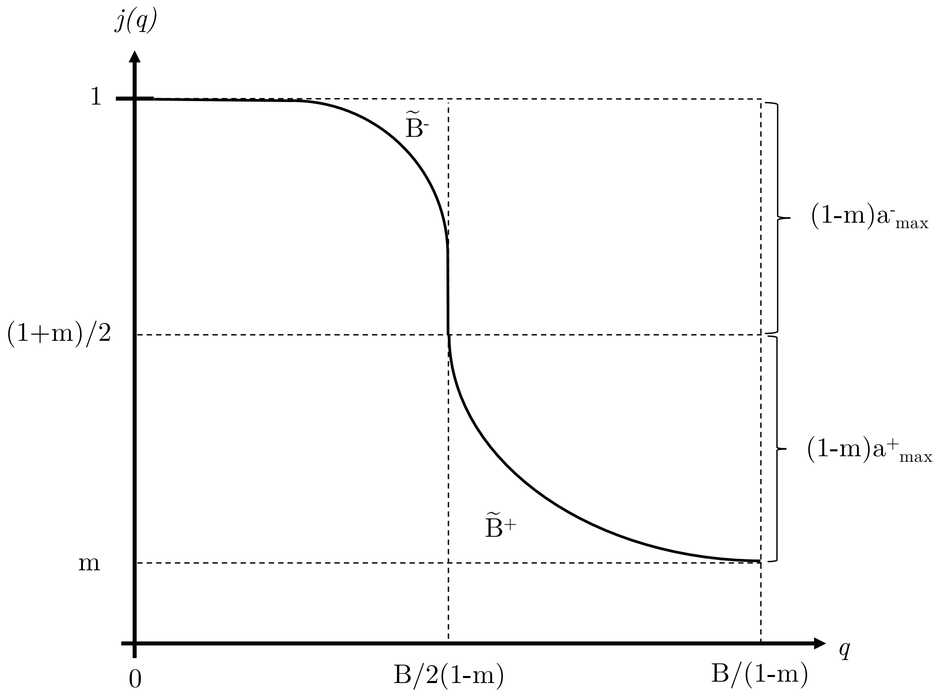


Figure 3: Illustration of asymmetric SBP markets.

and negative SBP market do not cancel each other out and the position of the interval $[c_0^-, c_1^+]$ has an impact on the total costs $C$ in (14). However, $c_0^-$, $c_1^- = c_0^+$ and $c_1^+$ do not depend on $\gamma^+$ and $\gamma^-$.[14] Therefore, the market equilibrium with asymmetric SBP markets cannot guarantee efficiency. Yet, the results of the parametrized model with asymmetric SBP markets for the German SBP market of 2015 only slightly differ from the corresponding symmetric model because the asymmetry is small (see following sections). We use this as a justification of our theoretical model in which we restrict our analysis to symmetric SBP markets.

### 5.2. Parametrization of the Model

We calibrate our model with the characteristics of the German wholesale market and SBP markets in 2015. Note that the results are presented in one hour with unit Euro/MWh, and

Austrian and German TSOs procure a common SBP merit-order, i.e., activation of SBP is linked within the two countries. Therefore, an analysis of the year 2016 had to include also the Austrian supply side.

[14]The market equilibrium depends on $B^+, B^-, \gamma^+, \gamma^-$, $a^+_{max}$, and $a^-_{max}$ but none of the conditions presented in Section 3.3 depend on $\gamma^+$ or $\gamma^-$.

that the current German positive and negative SBP markets are spilt into two time periods (regelleistung.net, 2017): a main period (peak) and a sub-period (offpeak).[15]

*5.2.1. Demand*

The total demand on the wholesale market was 525,000 GWh (BDEW and BMWi, 2016), which yields an average demand of $D = 59.93$ GW. The average demand on the positive and negative SBP market were $B^+ = 2.053$ GW and $B^- = 2.027$ GW and the average percentage demand were $\gamma^+ = 0.078$ and $\gamma^- = 0.060$ (BNetzA and BKartA, 2017). This yields average activated SBP capacities of $\tilde{B}^+ = 0.160$ GW and $\tilde{B}^- = 0.122$ GW.

*5.2.2. Market Characteristics*

The activated BE was 2,500 GWh in 2015. Hereof, 1,400 GWh were utilized for positive SBP and 1,100 GWh for negative SBP (BNetzA and BKartA, 2017). Thus, the demand for BE is not symmetric and positive SBP is used more frequently (56%) than negative SBP (44%). This yields maximum calling probabilities of $a^+_{max} = 0.56$ and $a^-_{max} = 0.44$.

The calling probabilities of 2015 are shown in Figure 4. Here, the interval [-2,000, 0] MW belongs to the negative SBP market and the interval $[0, 2{,}000]$ MW to the positive SBP market. We approximate these calling probabilities by the exponential function

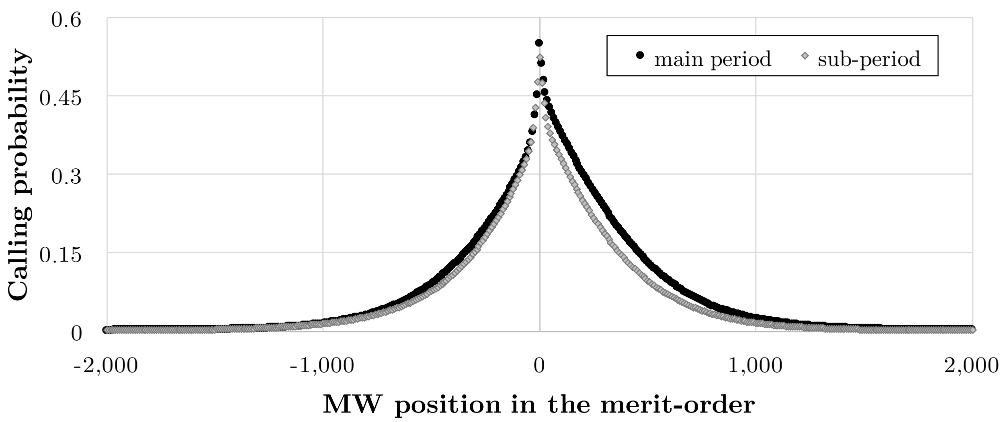


Figure 4: Empirical functions for the calling probabilities in the German positive (interval [0, 2,000] MW) and negative (interval [-2,000, 0] MW) SBP market (main period and sub-period) (50hertz, 2017).

$$a(x(c)) = a_{max} \frac{e^{-tx(c)} - e^{-tx(c_1)}}{e^{-tx(c_0)} - e^{-tx(c_1)}} = a_{max} \frac{e^{-tx(c)} - e^{-t}}{1 - e^{-t}} . \tag{18}$$

This represents the normalized merit-order with $x(c) = \frac{c-c_0}{c_1-c_0} \in [0, 1]$ for all $c \in [c_0, c_1]$, and

[15]The main period includes Monday to Friday 8am to 8pm (60 hours per week), and the sub-period (offpeak) includes Monday to Friday 8am to 8pm as well as Saturday and Sunday (108 hours per week).

gradient parameter $t$. Integrating $a(x(c))$ yields the percentage BE demand

$$\gamma = \int_0^1 a_{max} \frac{e^{-tx(c)} - e^{-t}}{1 - e^{-t}} \, dx = a_{max} \frac{1 - (t+1)\, e^{-t}}{t\,(1 - e^{-t})} \, .$$

Function (18) for the positive SBP market with $\gamma^+ = 0.078$, $a^+_{max} = 0.56$, $t^+ = 7.1$, and for the negative SBP market with $\gamma^- = 0.060$, $a^-_{max} = 0.44$, $t^- = 7.2$ are shown in Figure 5.

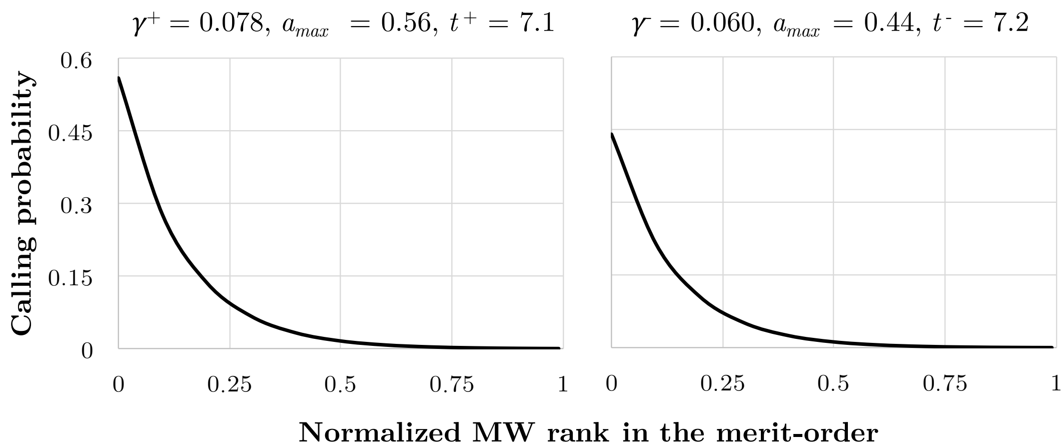


Figure 5: Approximation of the calling probabilities in the German positive and negative SBP market.

The (normalized) position in the merit-order $X(c)$ is given by

$$X(c) = \frac{\int_c^{c_1} x \, a(x(c)) \, dx}{\int_c^{c_1} a(x(c)) \, dx} \quad \forall c \in [c_0, c_1] \, ,$$

which yields the (normalized) BE price $P(c)$

$$P(c) = (1 - \vartheta)\, X(c) + \vartheta \quad \forall c \in [c_0, c_1] \, .$$

The pricing rule in the German SBP market is PaB not UP. Since there is no publicly available information about the extent of the markups on the variable costs under PaB, we neglect markups, i.e., $\vartheta = 0$. Thus, our calculation represents a lower bound for the BE costs. This yields $X(c) = P(c) = 0.14$. Regarding must-run capacities, we assume $m = 0.5$ for all types of power plants (Steck and Mauch, 2008; Just and Weber, 2008; Hundt et al., 2009).

*5.2.3. Supply Characteristics*

For the wholesale market we refer to the Intraday Continuous Auction with an the average price $p_S = 33.31$ Euro/MWh in 2015 (Fraunhofer ISE, 2017).[16] We calibrate our model by this price $p_S$. We also consider renewable energy sources, which typically have variable cost close to zero and, thus, set the intercept of the supply function (Christoph et al., 2013; Nestle, 2014; Milojcic and Dyllong, 2016; Niedermeier et al., 2017). We use the official data on installed capacity provided by the German regulator (BNetzA, 2016).[17] This yields $S(c) = 1.369\,c + 14.345$ in Figure 6, which includes the calibration point (33.31 Euro/MWh, 59,930 MW).[18] Since 2017, the German TSOs publish the prequalified capacities: for positive SBP 22.32 GW and 22.38 GW for negative SBP (regelleistung.net, 2017). Thus, we set $\delta = 0.22$.[19]

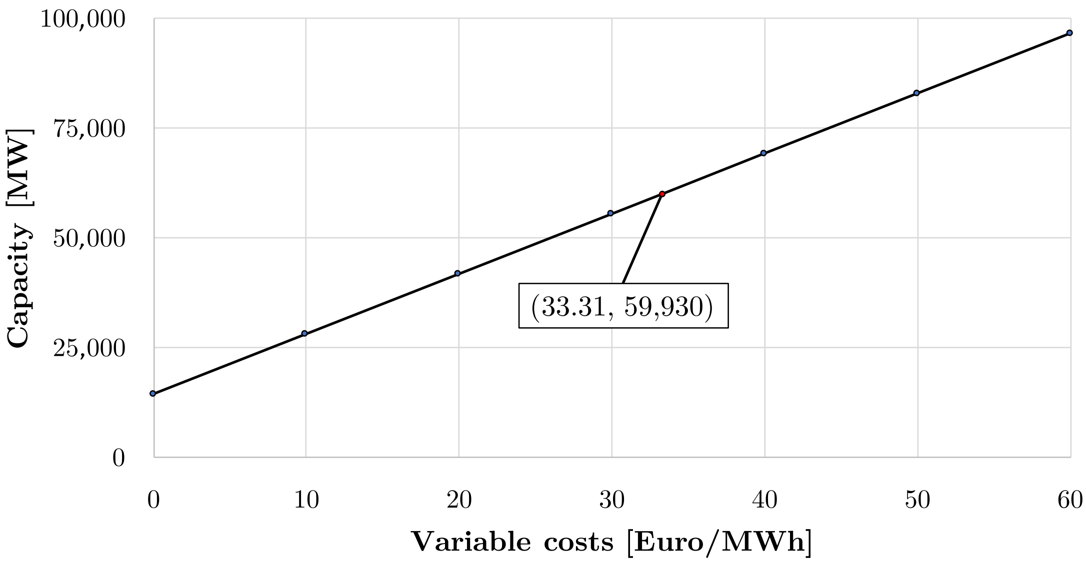


Figure 6: Linear approximation of the supply.

[16]The average price on the German Day Ahead Auction was 31.20 Euro/MWh, and 33.09 Euro/MWh on the Intraday Auction in 2015 (Fraunhofer ISE, 2017).

[17]According to BNetzA (2016), the information includes: "The Bundesnetzagentur's list of power plants includes existing power plants in Germany with a net nominal electricity capacity of at least 10 MW. It also includes capacities feeding into the German grid from Luxembourg, Switzerland and Austria. In addition, the list shows generation facilities of less than 10 MW which are eligible for receiving payment under the Renewable Energy Sources Act (EEG), grouped by federal state and energy source. Generation facilities under 10 MW not eligible for EEG are grouped by energy source." These also include wind onshore and offshore and solar power plants. Since their production strongly depends on the weather conditions, we use their actual energy production (BDEW, 2016) instead of their installed capacity.

[18]Note that a linear supply function is an assumption in our model. It does not fully reflect the real-life market characteristics and should only be seen as an approximation of the actual supply function.

[19]We do not differentiate with respect to the prequalified capacities per class of power plants, since it would not allow for a homogeneous $\delta$.

### 5.3. Results of the Integrated Model

The model results are presented in Tables 1 and 2.[20] The cost interval for the merit-order in the negative SBP market is [13.03, 26.49] Euro/MWh and [26.49, 40.13] Euro/MWh in the positive SBP market.[21] All suppliers of negative SBP and about half of the suppliers of positive SBP have lower variable cost than $p_S = 33.31$ Euro/MWh. The BP price in the positive SBP market is $p^+_{BP} = 6.82$ Euro/MWh, while the BP price in the negative SBP market equals zero. The average BE price in the positive market is $p^+_{BE} = 28.40$ Euro/MWh and the average BE price in the negative market is $p^-_{BE} = -24.61$ Euro/MWh. The profits in the merit-orders are $\pi(c^-_0) = 20.28$, $\pi(c^-_1 = c^+_0) = 6.82$, and $\pi(c^+_1) = 0.00$.[22]

| Parameter | Model GER 2015 |
|---|---|
| $c^-_0$ | 13.03 |
| $c^-_1 = c^+_0$ | 26.49 |
| $c^+_1$ | 40.13 |
| $p_S$ | 33.31 |
| $p^+_{BP}$ | 6.82 |
| $p^-_{BP}$ | 0.00 |
| $p^+_{BE}$ | 28.40 |
| $p^-_{BE}$ | -24.61 |

Table 1: Merit-order position and prices in the markets with unit Euro/MWh.

| Cost parameter | Model GER 2015 |
|---|---|
| Total system | 2,011,823 |
| Wholesale market | 1,996,268 |
| SBP market | 15,555 |
| Positive SBP: BP | 13,999 |
| Positive SBP: BE | 4,548 |
| Negative SBP: BP | 0 |
| Negative SBP: BE | -2,992 |

Table 2: Hourly cost of the power system with unit Euro/h.

The total costs of the power system are 2,011,823 Euro/h. Hereof, over 99% of the costs are due to the wholesale market, and less than 1% due to the SBP market (positive and negative).

[20] Note that the hourly cost of BE are 1,555 Euro. Thus, the BE cost that need to be considered by the suppliers are $dc = 0.026$ Euro/MWh.

[21] Note that we state the imputed cost $c + dc$. Further, the condition for $p_S$ holds (see Proposition 2).

[22] The model results for a symmetric German SBP market of 2015, i.e., $B^+ = B^- = 2.00$, $\gamma^+ = \gamma^- = 0.07$, and $a^+_{max} = a^-_{max} = 0.5$, are as follows: $dc = 0.09$, $c^-_0 = 13.38$, $c^-_1 = c^+_0 = 26.67$, $c^+_1 = 39.95$, $p^+_{BP} = 6.64$, $p^-_{BP} = 0.00$, $p^+_{BE} = 28.53$, $p^-_{BE} = -24.81$, $\pi(c^-_0) = 19.93$, $\pi(c^-_1 = c^+_0) = 6.64$, and $\pi(c^+_1) = 0.00$

The cost for positive SBP are 18,547 Euro/h and -2,992 Euro/h for negative SBP.[23]

### 5.4. Model Results and Empirical Market Results

The German regulator publishes annual data for the SBP market (BNetzA and BKartA, 2016, 2017). For comparing our modelled results of Section 5.3 with the actual data of 2013, 2014 and 2015, we convert our modelled results into annual values and calibrate these according to the conditions in 2015 (see Table 3).

| Cost parameter | Real 2013 | Real 2014 | Real 2015 | Model 2015 |
|---|---|---|---|---|
| **BP costs** | **345** | **210** | **141** | **123** |
| Costs pos. BP | 143 | 132 | 102 | 123 |
| Costs neg. BP | 202 | 78 | 39 | 0 |
| **BE costs** | **58** | **50** | **64** | **14** |
| Costs pos. BE | 95 | 65 | 72 | 40 |
| Costs neg. BE | -37 | -15 | -8 | -26 |
| **Total costs** | **403** | **260** | **205** | **137** |

Table 3: Empirical market results and asymmetric model results for the German SBP market of 2015 with unit Mio. Euro. Sources: regelleistung.net (2017); BNetzA and BKartA (2016, 2017).

Note that our parametrized model only accounts for the SBP costs of 2015. Yet, we consider it valuable to also illustrate prior market results. Since the German TSOs do not provide detailed annual costs, we estimate the cost parameters based on the market data provided by the TSOs (regelleistung.net, 2017). That is, we estimate the average BP costs per week and convert them into annual values.[24]

The costs for BE depend on two parameters: the BE bids and the actual demand for BE. On the one hand, the BE bids are very volatile, but on the other hand, if we assume for 2013

[23] The model results for a symmetric German SBP market of 2015 are as follows: Hourly total system costs: 2,010,075, hourly wholesale market costs: 1,996,268, hourly SBP costs 13,807, hourly positive BP costs: 13,286, hourly negative BP costs: 0, hourly positive BE costs: 3,994, hourly negative BE costs: -3,473.

[24] For our estimations, we use the following data:

| | | Positive SBP | | | Negative SBP | | |
|---|---|---|---|---|---|---|---|
| | | Avg. power bids [Euro/MW] | | Avg. power | Avg. power bids [Euro/MW] | | Avg. power |
| Year | Weeks | Main period | Sub period | demand [MW] | Main period | Sub period | demand [MW] |
| 2013 | 52 | 552 | 740 | 2,122 | 726 | 1,142 | 2,081 |
| 2014 | 52 | 456 | 782 | 2,058 | 268 | 491 | 1,987 |
| 2015 | 53 | 350 | 585 | 2,053 | 102 | 267 | 2,027 |

We multiplied the average power bids in the main and sub period with the number of weeks and with the average power demand for positive and negative SBP. Our estimates for the aggregated annual costs differ slightly from the official values: 353 Mio. Euro in 2013, 228 Mio. Euro in 2014, 155 Mio. Euro in 2015 (BNetzA and BKartA, 2016, 2017).

and 2014 the calling probabilities of 2015 (see Figure 5), then the average weighted BE bid of the activated plants is only slightly higher than the BE bid on the first position in the merit-order (less than 1 %). Thus, as an approximation, we estimate the BE costs by using the average bid on the first position in the merit-order and convert these to annual values.[25]

The empirical results illustrate that almost all costs continuously decreased. The total costs of 2013 (403 Mio. Euro) nearly halved in 2015 (205 Mio. Euro). The modelled result of 137 Mio. Euro indicates that there is still potential for further cost reductions.

The total BP costs decreased from 345 Mio. Euro in 2013 to 141 Mio. Euro in 2015 although the BP demand was nearly constant during these years (BNetzA and BKartA, 2016, 2017). The model predicts 123 Mio. Euro. For the BP costs in the positive and negative SBP market, the differences between the actual and the modelled results are larger. While our model predicts costs of 123 Mio. Euro for positive BP in 2015, the actual costs were 102 Mio. Euro. Remarkably, the actual costs are lower than the predicted costs. This indicates that suppliers understate their opportunity costs in their power bids.[26] Ocker et al. (2018b) offer an explanation: if the suppliers expect high energy prices, they submit low power bids in order to increase the probability of being awarded and, thus, to benefit from the high energy prices. That is, suppliers subsidize the power bid with the expected profits from the energy bids. The difference between the actual BE costs in the positive (negative) SBP market of 72 Mio. Euro (-8 Mio. Euro) and the prediction of 40 Mio. Euro (-26 Mio. Euro) gives support to this hypothesis. The BP costs in negative SBP market decreased more than 80% since 2013. In 2015 there is a difference of +39 Mio. Euro left between the actual and modelled results.

The development of the actual total BE costs is ambiguous and the development of the BE costs in the positive and negative SBP market differ. While BE costs in the positive SBP market decreased from 95 to 72 Mio. Euro (with a small increase from 2014 to 2015), the BE costs in the negative SBP market increased from -37 to -8 Mio. Euro.[27] The large difference between the total BE cost level (50 to 64 Mio. Euro) and the prediction of 14 Mio. Euro is mainly due to the large difference of the BE costs in the positive SBP market (72 and 40 Mio. Euro), which we discussed before.[28]

[25]The average BE bid for positive SBP in the main period (sub-period) on the first position in the merit-order with unit Euro/MWh was: 67.79 (64.02) in 2013, 54.26 (56.57) in 2014, and 51.98 (50.50) in 2015. The average BE bid for negative SBP in the main period (sub-period) on the first position in the merit-order with unit Euro/MWh was: -22.20 (-12.51) in 2013, -16.20 (-5.30) in 2014, and -11.61 (-5.25) in 2015. We multiplied these numbers with the assigned hours per week for the main period (60h) and the sub-period (108h), the number of weeks: 52 weeks in 2013 and 2014, and 53 weeks in 2015 as well as with the average energy demand for positive (negative) SBP with unit MW: 166 (264) in 2013, 133 (184) in 2014, and 160 (122) in 2015.

[26]Note that power plant dynamics may also impact the bidding strategy of suppliers. For example, some power plants may have capacity available as they are ramping down after a peak. This might reduce the opportunity cost of offering positive balancing capacity. For the sake of solvability and computability of our model, we do not consider such plant dynamics and the effects involved.

[27]Note that the BE demand declines since 2013 (BNetzA and BKartA, 2016, 2017).

[28]The modelled costs of the symmetric German SBP market of 2015 with unit Mio. Euro are as follows: 116

### 5.5. Balancing Energy Prices

In this section we discuss empirical energy bids and the impact of the BEPP on BE prices.

#### 5.5.1. Extreme Energy Bids

Empirical data reveal extreme bidding behavior, particularly on higher positions in the merit-orders. This result is supported by recent studies that find evidence for market imperfections in the German SBP markets (Heim and Götz, 2013; Ocker et al., 2018b; Ocker and Ehrhart, 2017; Ocker et al., 2018a). We illustrate the high energy bids for the merit-order positions 500, 1,000, and 1,500 of 2013, 2014, and 2015 in Table 4.

| | 2013 | | | 2014 | | | 2015 | | |
|---|---|---|---|---|---|---|---|---|---|
| | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 |
| Main period (pos.) | 103 | 164 | 242 | 94 | 164 | 349 | 84 | 159 | 447 |
| Sub-period (pos.) | 109 | 166 | 247 | 97 | 182 | 350 | 86 | 192 | 484 |
| Main period (neg.) | -8 | 19 | 123 | 27 | 111 | 497 | 38 | 155 | 723 |
| Sub-period (neg.) | 7 | 49 | 203 | 31 | 119 | 435 | 51 | 220 | 841 |

Table 4: Average energy bids with unit Euro/MWh for merit-order positions 500, 1,000, and 1,500 in the German SBP markets (main period and sub-period) of 2013, 2014, 2015. Source: regelleistung.net (2017)

In the negative SBP market, bids lower than zero have disappeared entirely from position 500 onwards since 2014. This means that suppliers offer negative BP if they are paid to reduce their load level.[29] Additionally, bids significantly increased over time on all positions. In the positive market, bids particularly increased on the last positions in the merit-order. Although the calling probability substantially decreases on higher positions (see Figure 4), the empirical data illustrate the potential effect of increasing bids on costs.

#### 5.5.2. Uniform Pricing and Length of the BEPP

The extremely high energy bids are accompanied by very low power bids (see Section 5.4). This reveals a major disadvantage of the applied scoring rule: since solely the power bid is relevant for winner determination, competition for energy bids is undermined, and facilitates the coordination on (extremely) high prices (Ocker et al., 2018b).

The European Commission (2017) implements UP in the future European SBP auction by arguing that it induces suppliers to report their true cost in their bids and, thus, leads to efficient auction outcomes. Ocker et al. (2018a) show that this reasoning is incorrect. In the

for positive BP, 0 for negative BP, 35 for positive BE, -30 for negative BE, resulting in total costs of 121.

[29]In the previous section we argued that this is a result of collusive behavior. However, it may also be seen against the background of dynamic effects: when reducing the power plant's load level, a supplier incurs costs if she wants to get back to her preferred output level (e.g., full load) in the subsequent period after providing negative BP. Suppliers may include these costs (partially) in their energy bids.

game-theoretical equilibrium of the one-shot auction, suppliers underbid their true cost in the energy bids. The reason for this is the inhomogeneity of the goods in the merit-order, i.e., the different calling probabilities. If UP is applied, the risk of setting the uniform price is low, and underbidding maximizes the expected profits in the one-shot auction because suppliers reach merit-order positions with higher calling probabilities (Ocker et al., 2018a).[30] Only if the BEPP is short, suppliers are incentivized to truthfully report their cost in their energy bids.[31] Therefore, we advocate to set the BEPP to a short value.

However, we doubt that a switch to UP will fundamentally change suppliers' bidding behavior because this seems to be guided by other principles. There is empirical evidence that suppliers abused their market power and coordinated (i.e., implicitly colluded) on high price levels (Heim and Götz, 2013; Ocker and Ehrhart, 2017). Changing the pricing rule will not impede collusion as long as the design elements, which facilitate collusion, are not affected: the regular repetition of the auction with a limited and stable set of suppliers.[32]

If UP is applied, the length of the BEPP directly impacts the BE costs: the longer the BEPP, the higher is the position in the merit-order that determines the uniform price. Our model allows the comparison of different BEPP figures. The results of the model in Section 5.4 refer to a value of $\vartheta = 0$, i.e., PaB without markups. If we set $\vartheta = 1$ (the longest BEPP), we find that the annual SBP costs rise to 149 Mio. Euro, i.e., an increase of 9%.[33]

## 6. Conclusion

This paper examines the interrelations of the electricity wholesale market and the SBP markets. Our analysis is based on the market interdependencies that BP suppliers cannot trade their capacities on the wholesale market, however, must run their plants at a minimal load. We develop an integrated market model for which we derive an equilibrium on all markets that ensures efficiency under certain assumptions. Our market setting relates to the recently discussed harmonization of the European BP markets. We prove that there exists an integrated market equilibrium that guarantees efficiency on the markets. By comparing the theoretical results with empirical German market data of 2015 we find discrepancies: our theoretical results predict lower total costs.

[30] Similar arguments hold for the position auctions of advertisements in search engines such as Google or Yahoo! (Varian, 2006; Edelman et al., 2007).

[31] If the BEPP is set to the smallest value, suppliers' payment for the energy bid is determined only by their energy bids, and therefore suppliers have an incentive to shade their bid as if PaB is applied (see Section 3.2).

[32] The European Commission (2017) also intends to introduce a voluntary energy bid market, i.e., suppliers that were not awarded within the regular SBP auction can submit an additional energy bid. This additional energy bid allows to be part of the merit-order without the power payment. This may increase the competition on the BE bids, and set an upper bound for energy bid payments (Ocker et al., 2018a).

[33] Setting $\vartheta = 1$ yields $P(c) = 1$. The costs for BP reduce to 107 Mio. Euro, whereas the costs for BE increase to 42 Mio. Euro. The reason for this is the reduction of the BP price to 5.96 Euro/MWh, and an increase of the BE price in the positive (negative) market to 39.46 Euro/MWh (-12.36 Euro/MWh).

Our market model is the first which considers the interdependencies of the electricity wholesale market and the BP markt within an analytical approach. Yet, there are three limitations that may reduce the external validity. First, we assume a linear supply function, which is a strong restriction. We are aware that this does not reflect the real supply characteristics. Second, our model considers the same share of BP power plants across the entire supply, which is a simplification of the market setting. Third, our assumption of homogeneous must-run capacities across all types of power plants is an approximation to actual power stations' characteristics. Here, a differentiation for several classes of power plants may be adequate.

Further research on BP markets could include experimental economics in a controlled laboratory setting. Here, the bidding behavior (in repeated auctions) could be investigated under different market designs. Finally, our integrated model could be applied to other European BP markets in order to evaluate auction outcomes more rigorously.

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## AppendixA. Proofs

*Proof of Proposition 1.* The existence of the equilibrium and 1., 2., and 3. follow by solving the equation system given by (M0), (M1), (M2), (S0), (S1), (S2), (S3) and $S(c) = \alpha\, c + \beta$. Alternatively, 3. follows directly from (M1).

*Proof of Proposition 2.* 1., 2., and 3. follow directly by solving the equation system given by (M0), (M1), (M2), (S0), (S1), (S2), (S3), symmetric BP markets and $S(c) = \alpha\, c + \beta$. The restriction $p_S \leq m\, c_1^+ + (1-m)\, c_0^+$ is implied by $\pi_S(c) - \pi_{BP}(c) \leq 0$, which yields

$$p_S - c \leq \frac{m}{1-m}(c_1^+ - p_S) + a(r(c))\, \vartheta\, (c_1^+ - c)\,.$$

Now consider $\vartheta = 0$, and the condition $c \geq c_0^+$. This yields

$$p_S \leq m\, c_1^+ + (1-m)\, c_0^+\,,$$

which is a sufficient condition for the claim.

*Proof of Proposition 3.*

1. The derivative of (7) is

$$\frac{\partial\, \pi_{BP}^+(c)}{\partial\, c} = -m + (1-m)\, (a'(r(c))\, r'(c)\, \vartheta\, (c_1^+ - c) - a(r(c))\, \vartheta) < 0\,,$$

   and is strictly decreasing in c, with $a'(r(c)) < 0$, $r'(c) > 0$. Differentiation (8) yields

$$\frac{\partial\, \pi_{BP}^-(c)}{\partial\, c} = -1 + (1-m)[a'(r(c))\, r'(c)\, \vartheta\, (c - c_0^-) + a(r(c))\, \vartheta] < 0\,,$$

   and is strictly decreasing in c, with $a'(r(c)) < 0$, $r'(c) > 0$.
2. Equation (6) directly implies that $\pi_S \geq 0$ for all $c \in [\underline{c}, p_S]$. In the positive market, reformulating (7) with $p_{BP}^+ = \frac{m}{1-m}(c_1^+ - p_S)$ immediately yields the result. For a stable market equilibrium, (M2) demands that $\pi_{BP}^-(c_1^-) = \pi_{BP}^+(c_0^+)$, with $\pi_{BP}^+(c_0^+) \geq 0$. Since the profits of all BP suppliers decrease in c, all profits in the negative BP market must be (weakly) greater than zero.
3. Consider the suppliers with variable cost $c \in [c_0^-, c_1^-]$. The proposition demands $\pi_S(c) - \pi_{BP}(c) \leq 0$, which is immediately implied by straightforward computation. For the suppliers with variable cost $c \in [c_0^+, c_1^+]$, see the proof of Proposition 2.

*Proof of Proposition 4.* The derivative of (14) with respect to $q_0$ is

$$\frac{\partial C}{\partial q_0} = -S_{nBP}^{-1}\left(D - \frac{1+m}{1-m}\,\frac{B}{2} - q_0\right) + S_{BP}^{-1}(q_0) + \int_0^{\frac{B}{1-m}} j(q)\,S_{BP}^{\prime -1}(q_0+q)\,dq$$

$$= -\frac{D - \frac{B}{2}\,\frac{1+m}{1-m} - q_0 - \beta\,(1-\delta)}{\alpha\,(1-\delta)} + \frac{q_0 - \delta\,\beta}{\delta\,\alpha} + \frac{B\,(1+m)}{2\,\alpha\,\delta\,(1-m)} \overset{!}{=} 0\,,$$

which yields

$$q_0^* = D\,\delta - \frac{B\,(1+m)}{2\,(1-m)}\,.$$

Reformulating shows that $c_0^{*-}$ equals the equilibrium $c_0^-$ in Proposition 2

$$c_0^{*-} = \frac{D-\beta}{\alpha} - \frac{\frac{B}{2}\,\frac{1+m}{1-m}}{\delta\,\alpha}\,.$$

*Proof of Proposition 5.* By Proposition 1, $p_s = \frac{D-\beta}{\alpha}$ in the equilibrium, which implies

$$D - q_{BP} - q_0 = (1-\delta)D\,. \tag{A.1}$$

By (13) and (14) ,

$$q_{BP} = J(m,B) = \frac{B(1+m)}{2(1-m)}\,. \tag{A.2}$$

Thus, by (A.1) and (A.2),

$$q_0 = \delta D - q_{BP} = \delta D - \frac{B(1+m)}{2(1-m)}\,, \tag{A.3}$$

$$\frac{\partial q_0}{\partial \delta} = D\,. \tag{A.4}$$

With (16), (17), and (A.1) the total costs (14) can be written as

$$C = C_{nBP} + C_{BP} + C_J \,,$$

$$C_{nBP} = \int\limits_{(1-\delta)\beta}^{(1-\delta)D} \frac{q}{(1-\delta)\alpha} - \frac{\beta}{\alpha} dq \,, \tag{A.5}$$

$$C_{BP} = \int\limits_{\delta\beta}^{q_0} \frac{q}{\delta\alpha} - \frac{\beta}{\alpha} dq \,, \tag{A.6}$$

$$C_J = \int\limits_{0}^{\frac{B}{1-m}} j(q) \left( \frac{q+q_0}{\delta\alpha} - \frac{\beta}{\alpha} \right) dq \,. \tag{A.7}$$

We have to prove

$$\frac{\partial C}{\partial \delta} = \frac{\partial C_{BP}}{\partial \delta} + \frac{\partial C_{nBp}}{\partial \delta} + \frac{\partial C_J}{\partial \delta} < 0 \,. \tag{A.8}$$

With (2) and (A.2) – (A.7),

$$\frac{\partial C_{nBP}}{\partial \delta} = -\frac{(D-\beta)^2}{2\alpha} \,,$$

$$\frac{\partial C_{BP}}{\partial \delta} = \frac{(D-\beta)^2}{2\alpha} - \frac{q_{BP}^2}{2\delta^2\alpha} \,,$$

$$\frac{\partial C_J}{\partial \delta} = \int\limits_{0}^{\frac{B}{1-m}} j(q) \left( \frac{q_{BP}-q}{\delta^2\alpha} \right) dq = \frac{q_{BP}^2}{\delta^2\alpha} - \frac{1}{\delta^2\alpha} \int\limits_{0}^{\frac{B}{1-m}} j(q) q \, dq \,,$$

which in (A.8) together with (A.2) yield

$$\frac{\partial C}{\partial \delta} = \frac{1}{\delta^2\alpha} \left( \frac{q_{BP}^2}{2} - \int\limits_{0}^{\frac{B}{1-m}} j(q) q \, dq \right) = \frac{1}{\delta^2\alpha} \left( \frac{B^2(1+m)^2}{8(1-m)^2} - \int\limits_{0}^{\frac{B}{1-m}} j(q) q \, dq \right) < 0 \,. \tag{A.9}$$

To prove (A.9) we consider the $j$-function

$$j_0(q) = \begin{cases} 1 & : q \in [0, \frac{B}{2(1-m)}) \\ m & : q \in [\frac{B}{2(1-m)}, \frac{B}{1-m}] \,, \end{cases} \tag{A.10}$$

which fulfils the requirements of a $j$-function (9) and $J(m, B) = \frac{B(1+m)}{2(1-m)}$. Since (A.10) assigns the highest possible $j$-value (i.e., 1) to the low $q$-values in $[0, \frac{B}{2(1-m)})$ and the lowest possible $j$-value (i.e., $m$) to the high $q$-values in $[\frac{B}{2(1-m)}, \frac{B}{1-m}]$, (A.10) is the $j$-function with lowest integral $\int_0^{\frac{B}{1-m}} j(q)q\,dq$ in the class of $j$-functions (9) with $J(m,B) = \frac{B(1+m)}{2(1-m)}$. That is, for all $j$-functions $j(q)$ in this class:

$$\int\limits_0^{\frac{B}{1-m}} j(q)q\,dq \geq \int\limits_0^{\frac{B}{1-m}} j_0(q)q\,dq = \frac{B^2(1+3m)}{8(1-m)^2}\,. \tag{A.11}$$

Since

$$\frac{B^2(1+3m)}{8(1-m)^2} > \frac{B^2(1+m)^2}{8(1-m)^2} \;\Leftrightarrow\; 1 > m > 0\,, \tag{A.12}$$

Condition A.9 is fulfilled and, thus, also Condition (A.8), which completes the proof.